

**RAMIFICATION & HOPF-GALOIS MODULE THEORY**  
**MAY 27–31, 2013**  
**DURHAM SCIENCE CENTER ROOM 287**

SCHEDULE

**Monday.**

- 9:00am** Byott: *Ramification, Galois module structure (and Hopf algebras)* (50 Minutes)  
**11:00am** Koch: *Breuil-Kisin modules in all characteristics* (50 Minutes)  
**1:00pm** Elder: *Geometric vs. arithmetic results in characteristic zero* (25 Minutes)  
**5:00pm** Memorial Day Party *chez* Elder. Incl. vegan options, and a number of Omaha friends (incl. kids).

**Tuesday.**

- 9:00am** *Welcome*, Dave Booker (Dean, College of Arts & Sciences)  
**9:10am** Childs: *Purely inseparable field extensions* (50 Minutes)  
**11:00am** Byott: *Scaffolds and Generalized Galois Module Structure* (25 Minutes)  
**1:00pm** Marklove: *Local Galois Module Structure in Characteristic  $p$*  (50 Minutes)

**Wednesday.**

- 9:00am** Keating *The Field of Norms* (50 Minutes)  
**11:00am** Byott: *Nilpotent (and soluble?) Hopf-Galois structures* (50 Minutes)  
**1:00pm** Truman: *Integral Hopf-Galois structures for tame extensions* (50 Minutes)  
**2:00pm** Childs: *Hopf Galois structures on Galois extensions of fields of order  $mp$ ,  $m < p$*  (25 Minutes)

**Thursday.**

- 9:00am** *Welcome*, Scott Snyder (Associate Vice Chancellor for Research & Creative Activity)  
**9:10am** Miller: *Perfectoid spaces* (50 Minutes)  
**11:00am** Keating: *Indices of Inseparability and New Ramification Breaks* (50 Minutes)  
**1:00pm** Elder: *Truncated exponential: Refined ramification, Galois scaffolds and beyond* (50 Minutes)

**Friday.**

- 9:00am** Underwood: *Extensions of Group Schemes in Characteristic  $p$*  (50 Minutes)  
**11:00am** Truman: *Hereditary associated orders for Hopf-Galois structures on tame extensions* (25 Minutes)

## ABSTRACTS

### **Nigel Byott.**

*Ramification, Galois module structure (and Hopf algebras).* 50 Minutes

Abstract: I will give a survey of some of the connections between ramification and Galois module structure, mainly in the context of local fields. I will also explain why it is natural to consider Hopf algebras and Hopf orders in this context.

*Scaffolds and Generalized Galois Module Structure.* 25 Minutes

Abstract: I will present recent (and on-going) joint work with Griff Elder about Galois scaffolds and their generalization to the non-Galois situation. When such a scaffold (of high enough "tolerance") exists, this in principle enables us to answer, purely from the numerical parameters of the scaffold, questions such as whether a given ideal is free over its associated order.

*Nilpotent (and soluble?) Hopf-Galois structures.* 50 Minutes

Abstract: A finite Galois extension of fields may admit various Hopf-Galois structures, to each of is associated a "type" (i.e. an isomorphism class of group of the same order as the Galois group). I will show how counting the Hopf-Galois structures of nilpotent type on a nilpotent extension can be reduced to the case of extensions of  $p$ -power degree. I will also sketch a proof of that Hopf-Galois structures of nilpotent type can only occur for soluble extensions.

### **Lindsay Childs.**

*Purely inseparable field extensions.* 50 Minutes

Abstract: The subject of purely inseparable field extensions was in fashion between 1930 and 1950, highlighted by Jacobson's exponent one Galois theory, and again in the period between 1968 and 1975 with attempts to generalize Jacobson's theory to more general finite purely inseparable extensions. This talk is an overview of some of this work, centered around Sweedler's 1968 Annals paper.

*Hopf Galois structures on Galois extensions of fields of order  $mp$ ,  $m < p$ .* 25 Minutes

Abstract: Many Galois extensions  $L/K$  of fields have Hopf Galois structures on them other than that given by the action by the Galois group  $G$ . The Hopf Galois structures on  $L/K$  correspond by Galois descent to regular subgroups of  $\text{Perm}(G)$  normalized by the image  $\lambda(G)$  of the left regular representation of  $G$  in  $\text{Perm}(G)$ . But most of the work on counting or classifying Hopf Galois structures has utilized a method of determining regular embeddings of  $G$  into  $\text{Hol}(N)$  for  $N$  a group of the same cardinality as  $G$ . This method has the disadvantage that it is difficult to describe either the Hopf algebra  $H$  acting on  $L/K$  or the action of  $H$  on  $L$ . But in a forthcoming paper, Tim Kohl has shown how one can classify Hopf Galois structures on  $L/K$  by directly obtaining regular subgroups of  $\text{Perm}(G)$  normalized by  $\lambda(G)$  when the order of  $G$  is  $mp$  where  $p$  is prime and  $p > m$ . We describe Kohl's approach, apply it to the case when  $G$  is the semidirect product of cyclic groups of orders  $m$  and  $p$ , and make some connections with previous work that used  $\text{Hol}(N)$ .

**Griff Elder.**

*Geometric vs. arithmetic results in characteristic zero.* 25 Minutes

Abstract: The topics of this week's conference, Ramification, Galois module structure and Hopf orders over a finite extension  $K$  of the  $p$ -adic numbers, exhibit radically different behavior when the absolute ramification is large rather than small relative to other invariants. Thinking of the situation when absolute ramification is large as "coming from" characteristic  $p$  can be illuminating. In this talk, through the presentation of a number of examples, I will explain a "philosophy/mind-set" that I have found very helpful.

*Truncated exponential: Refined ramification, Galois scaffolds and beyond.* 50 Minutes

Abstract: This talk is about a love affair with the polynomial

$$(1 + X)^{[Y]} = \sum_{i=0}^{p-1} \binom{Y}{i} X^i \in \mathbb{Z}_{(p)}[X, Y],$$

(actually joint work with Nigel Byott) and how sometimes you get more than you give.

**Kevin Keating.**

*The Field of Norms.* 50 Minutes

Abstract: Let  $K$  be a local field. For certain infinite extensions  $L/K$  Wintenberger constructs the "field of norms"  $X(L/K)$ , a local field of characteristic  $p$ . In this expository talk we outline the construction of  $X(L/K)$  and describe how it is related to groups of power series with the operation of composition.

*Indices of Inseparability and New Ramification Breaks.* 50 Minutes

Abstract: Let  $K$  be a finite extension of the  $p$ -adic field  $Q_p$  and let  $L/K$  be a totally ramified  $(Z/pZ)^2$ -extension which has a single ramification break  $b$ . Byott and Elder defined a "new ramification break"  $b_*$  for  $L/K$ . In this talk we consider the relationship between this new ramification break and the indices of inseparability of  $L/K$ , as defined by Fried and Heiermann. In particular, we show that if  $p > 2$  and the index of inseparability  $i_1$  of  $L/K$  is not equal to  $(p^2 - p)b$  then  $b_* = i_1 - (p^2 - p - 1)b$ .

**Alan Koch.**

*Breuil-Kisin modules in all characteristics.* 50 Minutes

Abstract: Let  $R$  be a discrete valuation ring. We give a definition of Breuil-Kisin modules which correspond to  $R$ -Hopf algebras which is independent of the characteristic of  $R$ . We then relate the characteristic zero and characteristic  $p$  cases with examples, showing how the characteristic  $p$  case arises naturally as a limit of the characteristic zero case.

**Maria Marklove.**

*Local Galois Module Structure in Characteristic  $p$ .* 50 Minutes

Abstract: For a finite, totally ramified Galois extension  $L/K$  (of degree  $p$ ) of local fields of characteristic  $p$ , we investigate the embedding dimension and the minimal number of generators for the associated order of the maximal ideals of  $L$ . We also obtain necessary and sufficient conditions for the freeness of these ideals over their associated orders.

**Daniel Miller.**

*Perfectoid spaces.* 50 Minutes

Abstract: Recently Peter Scholze was able to prove a wide range of special cases of Deligne's weight-monodromy conjecture by introducing a class of rigid-analytic objects called perfectoid spaces. I will describe a motivating theorem (due to Fontaine and Wintenberger) and the basic theory of perfectoid spaces.

**Paul Truman.**

*Integral Hopf-Galois structures for tame extensions.* 50 Minutes

Abstract: If  $L/K$  is a finite extension of  $p$ -adic fields which is at most tamely ramified and  $H = L[N]^G$  is a Hopf algebra giving a Hopf-Galois structure on the extension then we have identified a number of cases in which the associated order of  $\mathfrak{D}_L$  is the order  $\mathfrak{D}_L[N]^G$ . In this talk, we shall give criteria for  $\mathfrak{D}_L[N]^G$  to be a Hopf order in  $H$  and for  $\mathfrak{D}_L[N]^G$  to give a Hopf-Galois structure on the extension of rings  $\mathfrak{D}_L/\mathfrak{D}_K$ .

*Hereditary associated orders for Hopf-Galois structures on tame extensions.* 25 Minutes

Abstract: Let  $L/K$  be a finite Galois extension of  $p$ -adic fields such that  $p \nmid [L : K]$ , and  $H = L[N]^G$  a Hopf algebra giving a Hopf-Galois structure on the extension. If  $H$  is commutative, then  $\mathfrak{D}_L$  is free over its associated order in  $H$ . We investigate approaches to generalizing this result to noncommutative Hopf algebras.

**Rob Underwood.**

*Extensions of Group Schemes in Characteristic  $p$ .* 50 Minutes

Abstract: Let  $p$  be a rational prime, let  $n \geq 1$  be an integer and let  $\mathbb{F}_q$  denote the field with  $q = p^n$  elements. Let  $t$  be an indeterminate and let  $R = \mathbb{F}_q[[t]]$  and  $K = \text{Frac}(R) = \mathbb{F}_q((t))$ . Let  $C_p$  denote the cyclic group of order  $p$  generated by  $g$ . For each integer  $i \geq 0$ ,  $H(i) = R[\frac{g-1}{t^i}]$  is an  $R$ -Hopf order in  $KC_p$  corresponding to the group scheme  $\mathbb{G}_i = \text{Spec } H(i)$ . In this talk we investigate the extensions of  $\mathbb{G}_j$  by  $\mathbb{G}_i$ .

## 1. ADDITIONAL INFORMATION

**Design of conference.** Since 2:00pm in Omaha is 8:00pm in England, for our English guests I've scheduled everything during the first part of the day. I've also included empty hours between talks, so that there is plenty of time to discuss and ask questions. I would like to view this week as more of a workshop than a conference. Ideally, we make some connections and get some things figured out while we are all together (and away from the normal distractions of life). Ideally, we get some work done!

**Misc. information.** There is some information that you will want to have while in Omaha: Restaurants, directions to campus, places of interest, running trails, etc. When I see you at the airport, I will give you a packet of such info.

**Lecture space.** The talks will be in DSC 287. There is a computer projector, along with a side blackboard. So you can use both at the same time.

**Memorial Day Details.** Address is 5624 Leavenworth Street (phone: 402-553-5596), which is a pleasant walk from Scott Village. Maps will be provided, and I will pick up those who would prefer to drive. Part of the party is a celebration of the fact that Griff and artist friend Lori Elliot-Bartle are each one year older. There will be plenty of food, including vegan/vegetarian options. There will also be a number of Omaha friends. Weather permitting, the pool will be open. Bring a swim suit.

**Thursday Evening “Banquet”.** It would be nice to all go out to each other Thursday evening. We'll discuss this during the week, and make a decision then.