Hopf Galois Scaffolds on Purely Inseparable Extensions

Alan Koch

Agnes Scott College

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Outline

Let:

- L/K be an extension of (non archimedian) local fields of degree p^n, where p is the characteristic of the residue field of K.
- $\mathfrak{O}_K$ be the valuation ring of K
- $\mathfrak{O}_L$ be the valuation ring of L
- $\mathfrak{P}_L$ be the maximal ideal of L.

Suppose L/K is Galois, group G.
Integral Galois module theory seeks to describe all fractional ideals of L in terms of the G-action.
For $h \in \mathbb{Z}$, let

$$\mathfrak{A}_h = \{\alpha \in KG : \alpha(\mathfrak{P}_L^h) \subseteq \mathfrak{P}_L^h\}.$$

We can ask:

1. Is $\mathfrak{P}_L^h$ a free $\mathfrak{A}_h$-module?
2. In particular, is $\mathfrak{O}_L$ a free $\mathfrak{A} := \mathfrak{A}_0$-module?

Now suppose $K = \mathbb{F}_q((T)), p \mid q$ and L/K is purely inseparable.

Then L/K is not Galois, but L/K is Hopf Galois for numerous K-Hopf algebras H.

For a fixed H, let

$$\mathfrak{A}_h = \{\alpha \in H : \alpha(\mathfrak{P}_L^h) \subseteq \mathfrak{P}_L^h\}.$$

We will ask:

1. Does L possess an H-scaffold?
2. If so, does the scaffold allow us to determine integral Hopf Galois module structure?

We will see that the answers to both of these questions are "yes" for a certain class of Hopf algebras.

What is an *H*-scaffold?

It depends on ~~who~~ when you ask.

Write $L = K(x), x^{p^n} \in K, v_L(x) = -b < 0, p \nmid b$.

Let H be a Hopf algebra which makes L/K Hopf Galois.
Let $\mathfrak{T} > 1$ be an integer. Suppose we have $\{\Psi_s : 0 \leq s \leq n-1\} \subset H$ such that

$$\Psi_s(x^i) \equiv i_s x^{i-p^s} \bmod \mathfrak{P}_L^{\mathfrak{T}}$$

for $0 \leq i \leq p^n - 1$, where $i = i_0 + i_1 p + \cdots + i_{n-1} p^{n-1}$.

Then $\{\Psi_s\}$ gives rise to an *H*-scaffold of tolerance $\mathfrak{T}$ in the sense of [BCE].

Outline

Fix n. Pick r such that $r < n \leq 2r$, and pick $f \in K^\times$.

Define $H_{n,r,f} = K[t]/(t^{p^n})$ and

$$\Delta(t) = t \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell t^{p^r \ell} \otimes t^{p^r (p-\ell)},$$

where $w_\ell = (\ell!(p-\ell)!)^{-1}$.

Then $H_{n,r,f}$ is a local Hopf algebra with local dual (“local-local").

Also, $H_{n,r,f} \cong H_{n,r',f'}$ iff $r = r'$ and $f/f' \in (K^\times)^{p^{r+1}-1}$.

The map $\alpha : L \to L \otimes H$ given by

$$\alpha(x) = x \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell x^{p^r \ell} \otimes t^{p^r (p-\ell)}$$

gives L the structure of an $H_{n,r,f}$-comodule and makes L into an $H_{n,r,f}$-Galois object.

Outline

Pink = Previous

$$\Delta(t) = t \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell t^{p^r \ell} \otimes t^{p^r (p-\ell)}$$

$$\alpha(x) = x \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell x^{p^r \ell} \otimes t^{p^r (p-\ell)}$$

To construct a scaffold, we require an action, not a coaction.

Let $H = H^*_{n,r,f}$.

We will describe enough of H and its action on L to construct a scaffold.

$H = H^*_{n,r,f}$

Recall H has K-basis $\{z_j : 0 \le j \le p^n - 1\}$ with

$$z_j(x^i) = \delta_{i,j}.$$

Multiplication in H comes from comultiplication in $H^* = H_{n,r,f}$.
For example:

$$\begin{aligned} z_1^2(t^i) &= \mathrm{mult}(z_1 \otimes z_1)\Delta(t^i) \\ &= \mathrm{mult}(z_1 \otimes z_1)(t \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell t^{p^r \ell} \otimes t^{p^r(p-\ell)})^i \end{aligned}$$

Clearly, $z_1^2(t^i) = 0$ for $i \ne 2$ and $z_1^2(t^2) = 2$.

Thus, $z_1^2 = 2z_2$.

More generally, $z_1^j = j! z_j, 1 \leq j \leq p-1$.

Even more generally, $z_p^j = j! z_{jp}, 1 \leq j \leq p-1$.

Also, for $s < r$:

$$\begin{aligned} z_{p^s}^p(t^i) &= \text{mult}(z_{p^s}^{p-1} \otimes z_{p^s})\Delta(t^i) \\ &= \text{mult}((-1)z_{(p-1)p^s} \otimes z_{p^s})(t \otimes 1 + 1 \otimes t + f\sum_{\ell=1}^{p-1} w_\ell t^{p^r\ell} \otimes t^{p^r(p-\ell)})^i \\ &= 0 \end{aligned}$$

since it is impossible to get a nontrivial $t^{(p-1)p^s} \otimes t^{p^s}$ term.

Thus, $z_{p^s}^p = 0$.

More complicated for $s \geq r$.

For example,

$$z_{p^s}^p(t^{p^{s-r}}) = \text{mult}(z_{p^s}^{p-1} \otimes z_{p^s})\Delta(t^{p^{s-r}})$$

$$= \text{mult}((p-1)! z_{(p-1)p^s} \otimes z_{p^s})(t \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} w_\ell t^{p^r \ell} \otimes t^{p^r(p-\ell)})^{p^{s-r}}$$

$$= \text{mult}(-z_{(p-1)p^s} \otimes z_{p^s})(t^{p^{s-r}} \otimes 1 + 1 \otimes t^{p^{s-r}} + f^{p^{s-r}} \sum_{\ell=1}^{p-1} w_\ell t^{p^s \ell} \otimes t^{p^s(p-\ell)}),$$

and $t^{(p-1)p^s} \otimes t^{p^s}$ occurs when $\ell = p-1$.

Thus,

$$z_{p^s}^p(t^{p^{s-r}}) = \text{mult}(-z_{(p-1)p^s} \otimes z_{p^s}) f^{p^{s-r}} w_{p-1} t^{(p-1)p^s} \otimes t^{p^s} = f^{p^{s-r}}.$$

Generally, if $z^p_{p^s}(t^i) \neq 0$ then

$$\begin{aligned} i_1 + p^r\ell' &= (p-1)p^s \\ i_2 + p^r\ell'' &= p^s, \end{aligned}$$

where

$$\begin{aligned} i &= i_1 + i_2 + i_3 \\ i_3 &= i_{3,1} + i_{3,2} + \cdots + i_{3,p-1} \\ \ell' &= i_{3,1} + 2i_{3,2} + \cdots + (p-1)i_{3,p-1} \leq (p-1)i_3 \\ \ell'' &= (p-1)i_{3,1} + (p-2)i_{3,2} + \cdots + i_{3,p-1} = pi_3 - \ell' \\ (p-1)p^s &= i_1 + p^r\ell' \\ p^s &= i_2 + p^r\ell''. \end{aligned}$$

$$\left.\begin{array}{rcl} i_1 + p^r \ell' & = & (p-1)p^s \\ i_2 + p^r \ell'' & = & p^s \end{array}\right\} \text{Need to solve these.}$$

$$\begin{aligned} i &= i_1 + i_2 + i_3 \\ i_3 &= i_{3,1} + i_{3,2} + \cdots + i_{3,p-1} \\ \ell' &= i_{3,1} + 2i_{3,2} + \cdots + (p-1)i_{3,p-1} \leq (p-1)i_3 \\ \ell'' &= (p-1)i_{3,1} + (p-2)i_{3,2} + \cdots + i_{3,p-1} = pi_3 - \ell' \\ (p-1)p^s &= i_1 + p^r \ell' \\ p^s &= i_2 + p^r \ell'' \end{aligned}$$

- If $i < p^{s-r}$ then $z^p_{p^s}(t^i) = 0$.
- If $i = p^{s-r}$ then <u>the</u> solution to the system above is

$$\ell' = (p-1)p^{s-r}, \ell'' = p^{s-r}, i_1 = i_2 = 0, i_3 = p^{s-r+1}.$$

- If $p^{s-r} < i < p^{s-r+1}$ then there is no solution, i.e., $z^p_{p^s}(t^i) = 0$.
- If $i \geq p^{s-r+1}$ there can be multiple solutions.

Example

Let $p = 3, r = 2, s = 3, n = 4$.
It can be shown that $z_9^3 = fz_1$.
But,

$$z_{27}^3(x^3) = 2f^3$$
$$z_{27}^3(x^{29}) = 2f^2$$
$$z_{27}^3(x^i) = 0, i \neq 3, 29.$$

Thus,

$$\begin{aligned} z_{27}^3 &= 2f^3 z_3 + 2f^2 z_{29} \\ &= 2f^3 z_3 + 2f^2 z_2 z_{27} \\ &= 2f^3 z_3 + f^2 z_1^2 z_{27}. \end{aligned}$$

H has K-basis:

$$\{z_1^{j_0} z_p^{j_1} \cdots z_{p^{n-1}}^{j_{n-1}} : 0 \leq j \leq p-1\}.$$

Remarks:

- $\{z_1, \ldots, z_{p^{n-1}}\}$ is not a minimal generating set for H as a K-algebra.
- $\{z_{p^{n-r}}, \ldots z_{p^{n-1}}\}$ is a minimal generating set.
- The complete algebra structure for H is:

$$H = K[z_{p^{n-r}}, \ldots z_{p^{n-1}}]/(z_{p^{n-r}}^p, \ldots, z_{p^{r-1}}^p, z_{p^r}^{p^2}, \ldots, z_{p^{n-1}}^{p^2})$$

- The coalgebra structure is, as usual,

$$\Delta(z_j) = \sum_{i=0}^{j} z_i \otimes z_{j-i}, 0 \leq j \leq p^n - 1,$$

which can be rewritten in terms of the generators.

H has K-basis:

$$\{z_1^{j_0} z_p^{j_1} \cdots z_{p^{n-1}}^{j_{n-1}} : 0 \leq j_s \leq p-1\}$$

This is the description of H which will be most useful going forward.

Outline

$L = K(x), x^{p^n} \in K, v_L(x) = -b < 0, p \nmid b.$
$\alpha : L \to L \otimes H$ is given by

$$\alpha(x) = x \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} \frac{1}{\ell!(p-\ell)!} x^{p^r \ell} \otimes t^{p^r(p-\ell)}.$$

The action of H on L is

$$z_j(x^i) = \text{mult}(1 \otimes z_j)\alpha(x^i).$$

So, for example,

$$z_j(x) = \begin{cases} x & j = 0 \\ 1 & j = 1 \\ ((jp^{-r})!(p - jp^{-r})!)^{-1} f x^{p^{r+1}-j} & p^r \mid j \text{ and } j < p^{r+1} \\ 0 & \text{otherwise} \end{cases}$$

$L = K(x), x^{p^n} \in K, v_L(x) = -b < 0, p \nmid b.$

$$\alpha(x) = x \otimes 1 + 1 \otimes t + f \sum_{\ell=1}^{p-1} \frac{1}{\ell!(p-\ell)!} x^{p^r \ell} \otimes t^{p^r(p-\ell)}.$$

$$z_j(x^i) = \mathrm{mult}(1 \otimes z_j)\alpha(x^i).$$

Let

$$i = \sum_{s=0}^{n-1} i_s p^s.$$

Then:

$$z_{p^s}(x^i) = \begin{cases} i_s x^{i-p^s} & 0 \le s \le r-1 \\ i_r x^{i-p^r} - ifx^{p^r(p-1)+i-1} & s = r \\ \text{complicated} & s > r \end{cases}.$$

$$z_{p^s}(x^i) = \begin{cases} \underset{\sim\sim\sim}{i_s x^{i-p^s}} & 0 \le s \le r-1 \\ i_r x^{i-p^r} - i f x^{p^r(p-1)+i-1} & s = r \\ \text{complicated} & s > r \end{cases}$$

Example ($p = 3, r = 2, s = 3, n = 4$)

$i = 35 = 2(1) + 2(3) + 1(27) \Rightarrow i_0 = 2, i_1 = 2, i_2 = 0, i_3 = 1.$
This can be verified directly:

$$z_{27}(x^{35}) = \underset{\sim}{x^8} + 2f^2x^{60} + f^3x^{86}.$$

In this "complicated" case, we see an $i_s x^{i-p^s}$ term.

$$z_{p^s}(x^i) = \begin{cases} \boxed{i_s x^{i-p^s}} & 0 \le s \le r-1 \\ i_r x^{i-p^r} - ifx^{p^r(p-1)+i-1} & s = r \\ \text{complicated} & s > r \end{cases}$$

$v_L(x) = -b.$

1. For $s < r$ either $v_L(z_{p^s}(x^i)) = \infty$ or

$$v_L(z_{p^s}(x^i)) = v_L(i_s x^{i-p^s}) = -b(i - p^s) = v_L(x^i) + bp^s.$$

$$z_{p^s}(x^i) = \begin{cases} i_s x^{i-p^s} & 0 \le s \le r-1 \\ \boxed{i_r x^{i-p^r} - ifx^{p^r(p-1)+i-1}} & s = r \\ \text{complicated} & s > r \end{cases}$$

2. Either $v_L(z_{p^r}(x^i)) = \infty$ or

$$\begin{aligned} v_L(z_{p^r}(x^i)) &= v_L(i_r x^{i-p^r} - ifx^{p^r(p-1)+i-1}) \\ &\ge \min\{v_L(i_r x^{i-p^r}), v_L(fx^{p^r(p-1)+i-1})\} \\ &\ge \min\{-b(i-p^r), v_L(f) - b(p^r(p-1)+i-1)\}. \end{aligned}$$

If $v_L(f) > b(p^r(p-1)+i-1)) - b(i-p^r) = b(p^{r+1}-1)$ then the minimum is $-b(i-p^r)$.
For such an f we have $v_L(z_{p^r}(x^i)) = v_L(x^i) + bp^r$.

We will assume $v_L(f) > b(p^{r+1}-1)+1$.
(This is nice and legal since $H_{n,r,f} = H_{n,r,T^{p^{r+1}-1}f}$.)

Nice and legal

Let $H = H_{n,r,f} = K[t]/(t^{p^n}), L = K(x), x^{p^n} \in K$.

Let $u = T^{-1}t$. Then $H = K[u]/(u^{p^n})$ and

$$\begin{aligned}\Delta(u) &= T^{-1}\Delta(t) \\ &= T^{-1}t \otimes 1 + 1 \otimes T^{-1}t + T^{-1}f\sum_{\ell=1}^{p-1} t^{p^r\ell} \otimes t^{p^r(p-\ell)} \\ &= u \otimes 1 + 1 \otimes u + T^{-1}f\sum_{\ell=1}^{p-1}(Tu)^{p^r\ell} \otimes (Tu)^{p^r(p-\ell)} \\ &= u \otimes 1 + 1 \otimes u + T^{p^{r+1}-1}f\sum_{\ell=1}^{p-1} u^{p^r\ell} \otimes u^{p^r(p-\ell)}\end{aligned}$$

So $H = H_{n,r,T^{p^{r+1}-1}f}$.

We'll study this $u = T^{-1}t$ substitution more later.

$$z_{p^s}(x^i) = \begin{cases} i_s x^{i-p^s} & 0 \leq s \leq r-1 \\ i_r x^{i-p^r} - ifx^{p^r(p-1)+i-1} & s = r \\ \boxed{\text{complicated}} & s > r \end{cases}$$

3. Messier, but it can be shown that, with no further restriction on f, either $v_L(z_{p^s}(x^i) = \infty$ or

$$v_L(z_{p^s}x^i) = v_L(i_s x^{i-p^s}) = -b(i - p^s) = v_L(x^i) + bp^s.$$

Theorem

Every element of the set

$$\{z_{p^s} : 0 \leq s \leq n-1\}$$

satisfies

$$z_{p^s}(x^i) \equiv i_s x^{i-p^s} \bmod \mathfrak{P}_L^{v_L(f)-b(p^{r+1}-1)},$$

and hence $\{z_{p^s} : 0 \leq s \leq n-1\}$ *forms an H-scaffold of tolerance* $v_L(f) - b(p^{r+1}-1) > 1$.

$$z_{p^s}(x^i) \equiv i_s x^{i-p^s} \mod \mathfrak{P}_L^{v_L(f)-b(p^{r+1}-1)}$$

Let $\rho \in L$, $v_L(\rho) = b$.
For example, we could take $\rho = Tx^{p^n-1}$.
For $0 \le j \le n-1$, write $j = j_0 + j_1 p + \cdots + j_{n-1}p^{n-1}, 0 \le j_s \le p-1$.

Properties

1. $v_L(z_{p^s}(\rho)) = b(1 + p^s)$.
2. $v_L(z_1^{j_0} \cdots z_{p^{n-1}}^{j_{n-1}}(\rho)) = b(1 + j)$.
3. $\{\prod_{s=0}^{n-1} z_{p^s}^{j_s}(\rho) : 0 \le j_s \le p-1\}$ is a K-basis for L.
4. b is (the Hopf Galois analogue of) an integer certificate for this action.

Outline

Recall $K = \mathbb{F}_q((T))$ and

$$\mathfrak{A}_h = \{\alpha \in H : \alpha(\mathfrak{P}_L^h) \subseteq \mathfrak{P}_L^h\},$$

and we wish to know:

1. is $\mathfrak{P}_L^h$ a free $\mathfrak{A}_h$-module?
2. is $\mathfrak{O}_L$ a free $\mathfrak{A} := \mathfrak{A}_0$-module?

We have a scaffold $\{z_{p^s}\}$ of tolerance $v_L(f) - b(p^{r+1} - 1)$.

If $v_L(f) - b(p^{r+1} - 1) > 2p^n - 1$, then the scaffold allows us to determine integral Hopf Galois module structure.

From now on, assume $v_L(f) > 2p^n - 1 + b(p^{r+1} - 1)$.

This is still nice and legal.

Still nice and legal

Let $H = H_{n,r,f} = K[t]/(t^{p^n}), L = K(x), x^{p^n} \in K$.

We have a scaffold of tolerance $\mathfrak{T} = v_L(f) - b(p^{r+1} - 1)$.

Let $u = T^{-1}t$. Then $H = H_{n,r,T^{p^{r+1}-1}f}$.

Let $y_{p^s} = Tz_{p^s}$. Then

$$\begin{aligned} y_{p^s}(x^i) &\equiv i_s x^{i-p^s} \bmod \mathfrak{P}_L^{\mathfrak{T}'}, \\ \mathfrak{T}' &= v_L(T^{p^{r+1}-1}f) - b(p^{r+1} - 1) \\ &= p^n(p^{r+1} - 1) + \mathfrak{T}. \end{aligned}$$

So we have scaffolds of arbitrarily high tolerance.

For each $0 \leq j \leq p^n - 1, j = \sum_{s=0}^{n} j_s p^s, 0 \leq b - h \leq p^n - 1$, let

$$d_h(j) = \left\lfloor \frac{bj + b - h}{p^n} \right\rfloor$$

$$w_h(j) = \min\{d_h(i+j) - d_h(i) : 0 \leq i \leq p^n - 1,\ i_s + j_s \leq p - 1 \text{ for all } s\}.$$

Then, by [BCE],

1. $\mathfrak{A}_h$ has $\mathfrak{O}_K$-basis $\left\{T^{-w_h(j)} z_1^{j_0} z_p^{j_1} \cdots z_{p^{n-1}}^{j_{n-1}} : 0 \leq j \leq p^n - 1\right\}$.
2. $\mathfrak{P}_L^h$ is a free $\mathfrak{A}_h$-module if and only if $w_h(j) = d_h(j)$ for all $0 \leq j \leq p^n - 1$; if this equality holds then $\mathfrak{P}_L^h = \mathfrak{A}_h \cdot \rho,\ v_L(\rho) = b$.
3. $\mathfrak{O}_K$ is a free $\mathfrak{A}$-module if the least nonnegative residue of b divides $(p^m - 1)$ for some $1 \leq m \leq n$. If so, $\mathfrak{O}_L = \mathfrak{A} \cdot \rho$.
4. If $w_h(j) \neq d_h(j)$, then $\mathfrak{P}_L^h$ can be generated over $\mathfrak{A}_h$ using ℓ generators, where $\ell =$ $\left\{{}^{\#} i : d_h(i) > d_h(i-j) + w_h(j) \text{ for all } 0 \leq j \leq p^{n-1} \text{ with } j_s \leq i_s\right\}$

An example. Let $b = 1$.Then $2 - p^n \leq h \leq 1$ and

$$d_h(j) = \left\lfloor \frac{j+1-h}{p^n} \right\rfloor = \begin{cases} 1 & j \geq p^n - 1 - h \\ 0 & j < p^n - 1 - h \end{cases}$$

Since

$$w_h(j) = \min\{d_h(i+j) - d_h(i) : 0 \leq i \leq p^n - 1,\ i_s + j_s \leq p - 1 \text{ for all } s\},$$

we have $w_h(j) \leq d_h(j)$ for all j.

So $w_h(j) = d_h(j) = 0$ for all $j < p^n - 1 - h$.

It can be shown that $w_h(j) = 1$ for all other j if and only if $h > (1 - p^n)/2$, hence $\mathfrak{P}_L^h = \mathfrak{A}_h \cdot \rho$ iff $h > (1 - p^n)/2$.

So $\mathfrak{P}_L^h$ is free over $\mathfrak{A}_h$ if and only if $h > (1 - p^n)/2$.

In particular, $\mathfrak{O}_L$ is free over $\mathfrak{A}$.

This result can be adapted for general h.

What changes if we use a different Hopf algebra?

Almost nothing.

1. $\mathfrak{A}_h$ has $\mathfrak{O}_K$-basis $\left\{T^{-w_h(j)} z_1^{j_0} z_p^{j_1} \cdots z_{p^{n-1}}^{j_{n-1}} : 0 \le j \le p^n - 1\right\}$.
2. $\mathfrak{P}_L^h$ is a free $\mathfrak{A}_h$-module if and only if $w_h(j) = d_h(j)$ for all $0 \le j \le p^n - 1$; if this equality holds then $\mathfrak{P}_L^h = \mathfrak{A}_h \cdot \rho,\ v_L(\rho) = b$.
3. $\mathfrak{O}_K$ is a free $\mathfrak{A}$-module if the least nonnegative residue of b divides $(p^m - 1)$ for some $1 \le m \le n$. If so, $\mathfrak{O}_L = \mathfrak{A} \cdot \rho$.
4. If $w_h(j) \ne d_h(j)$, then $\mathfrak{P}_L^h$ can be generated over $\mathfrak{A}_h$ using ℓ generators, where $\ell =$ $\{\#i : d_h(i) > d_h(i-j) + w_h(j) \text{ for all } 0 \le j \le p^{n-1} \text{ with } j_s \le i_s\}$

The only changes are in the multiplication and action of the z_{p^s}'s.

Recall that $H = H_{n,r,f}$ can act on L in $p^{n-1}(p-1)$ ways.

What changes if we use a different action?

Again, almost nothing.

1. $\mathfrak{A}_h$ has $\mathfrak{O}_K$-basis $\left\{ T^{-w_h(j)} z_1^{j_0} z_p^{j_1} \cdots z_{p^{n-1}}^{j_{n-1}} : 0 \leq j \leq p^n - 1 \right\}$.
2. $\mathfrak{P}_L^h$ is a free $\mathfrak{A}_h$-module if and only if $w_h(j) = d_h(j)$ for all $0 \leq j \leq p^n - 1$; if this equality holds then $\mathfrak{P}_L^h = \mathfrak{A}_h \cdot \rho,\ v_L(\rho) = b$.
3. $\mathfrak{O}_K$ is a free $\mathfrak{A}$-module if the least nonnegative residue of b divides $(p^m - 1)$ for some $1 \leq m \leq n$. If so, $\mathfrak{O}_L = \mathfrak{A} \cdot \rho$.
4. If $w_h(j) \neq d_h(j)$, then $\mathfrak{P}_L^h$ can be generated over $\mathfrak{A}_h$ using ℓ generators, where $\ell =$ $\left\{ {}^{\#} i : d_h(i) > d_h(i-j) + w_h(j) \text{ for all } 0 \leq j \leq p^{n-1} \text{ with } j_s \leq i_s \right\}$

The only change is in the multiplication of the z_{p^s}'s.

Outline

Things to think about:

- Do Hopf Galois scaffolds exist for the more general class $H^*_{n,r,f}$ (i.e., for $d \geq 1$)?

- From an integral Galois module theory perspective, is there any reason to pick one Hopf algebra over another, or one action over another?

Thank you.

(Again.)