**RAMIFICATION AND GALOIS MODULE THEORY: OMAHA 2014**
**MAY 19–23, 2014**

## Schedule

**Monday.**

**9:00am** *Welcome*, Dave Boocker (Dean, College of Arts & Sciences)

**9:30am** Keating: *Parametrizing extensions of local fields.*

**11:00am** Huynh: *Enumerating $k$-isomorphism classes of totally ramified degree-$p$ extensions of the local field $k((\pi))$* (25 Minutes)

**1:00pm** Roberts *A database of number fields.* (25 Minutes)

**Tuesday.**

**9:30am** Elder: *Polishing an approach to two old problems.*

**11:00am** Byott : *Scaffolds and Bondarko diagrams.*

**1:00pm** Sundukova: *Local Galois module structure in prime characteristic and Galois scaffolds* (25 Minutes)

**1:00pm** Koch: *Hopf Galois structures on modular extensions.* (25 Minutes)

**Wednesday.**

**9:30am** Childs: *Abelian Hopf Galois structures on $C_p^n$-Galois extensions of fields.*

**11:00am** Truman: *Canonical nonclassical Hopf Galois module structure of nonabelian Galois extensions.*

**1:00pm** Roberts: *Primitive extensions of $\mathbb{Q}_p$ and trinomial-Eisenstein polynomials.* (25 Minutes)

**Thursday.**

**9:30am** Koch: *Hopf Galois scaffolds on purely inseparable extensions.*

**11:00am** Elder: *Hopf orders in elementary abelian group rings.*

**1:00pm** Underwood: *Duals of Hopf orders in characteristic $p$.* (25 Minutes)

**Friday.**

**9:30am** Truman *: Hopf Galois structures on quaternionic extensions and some examples of Martinet.*

**11:00am** Byott: *Quasitriangular structures on (dual) monoid rings.*

## Abstracts

**Nigel Byott.**

*Scaffolds and Bondarko Diagrams.* 50 Minutes

Abstract: This is a preliminary report on my attempts to relate two approaches to local Galois module structure, namely Galois scaffolds as developed by Griff Elder and myself, and Bondarko's theory of (semi-) stable extensions, defined in terms of "diagrams". Bondarko's set-up can be extended to the Hopf-Galois situation. For the action of a divided power Hopf algebra on a simple inseparable extension, both theories then give essentially the same thing. For extensions of the form $K(\sqrt[p^n]{a})/K$ for "nice" $a \in K$, Bondarko's approach leads to new examples of scaffolds. This works even if $K$ does not contain a primitive $p^n$th root of unity, giving scaffolds on these non-normal extensions in their "obvious" Hopf-Galois structure.

*Quasitriangular structures on (dual) monoid rings.* 50 Minutes

Abstract: Rob Underwood posed the problem of determining all quasitriangular structures on the monoid rings, or their duals, of finite monoids, in particular for the monoids arising from the algebraic version (due to Nichols and Underwood) of the Myhill-Nerode Theorem of theoretical computer science. I will show that, for a particularly simple infinite family of such monoids, both the monoid ring and its dual admit a unique quasitriangular structure.

**Lindsay Childs.**

*Abelian Hopf Galois structures on $C_p^n$-Galois extensions of fields.* 50 Minutes

Abstract: Let $L/K$ be a Galois extension of fields with Galois group isomorphic to $G = (\mathbb{F}_p^n, +)$. A neat thing about studying abelian Hopf Galois structures on $L/K$ is that those structures relate to commutative nilpotent $\mathbb{F}_p$-algebras of dimension $n$ and to certain $n$-dimensional degree 2 polynomial formal groups. We'll use these ideas to obtain asymptotic estimates on the number of Hopf Galois structures of type $G$ on $L/K$ as $n$ goes to infinity.

**Griff Elder.**

*Polishing an approach to two old problems.* 50 Minutes

Abstract: When things work out nicely, really nicely, there ought to be a perspective, a vantage point from which it all becomes simple – so simple as to possibly be trivial. The purpose of this expository talk is to: Recall some of the questions that drive our research and their accompanying thorny complications; List ideas (vantage points) that have added clarity, proven both evocative and useful; And then ask for more such ideas.

*Hopf orders in elementary abelian group rings.* 50 Minutes

Abstract: Let $K$ be a complete local field with perfect residue field of characteristic $p$, and let $G$ be a $p$-group. In 1970, Tate and Oort classified Hopf algebras over the valuation ring $\mathfrak{O}_K$ (i.e. Hopf orders) in $K[G]$ where $G$ has order $p$ . When $G$ has order $p^2$, the Hopf orders in $K[G]$ were classified separately by Greither (1992) and Byott (1993). Since then, there have been a number of papers by Underwood and Childs (including others) describing families of Hopf orders in $K[G]$ for $K$ of characteristic 0 and $G$ either elementary abelian or cyclic with $|G| = p^n$ and $n \geq 3$. These descriptions require the $p$th roots of unity to lie in $K$ and thus are intrinsically characteristic 0 in nature. Notably, the classifications for $n = 3$ remain incomplete. Motivated by the Hopf orders produced by Galois scaffolds, I will describe a characteristic independent approach to the description of Hopf orders: Letting $G$ be elementary abelian of degree $p^n$, Hopf orders are 1st determined in characteristic $p$, then lower bounds on the absolute ramification $v_K(p)$ are determined so that the result continues to hold in characteristic 0. The resulting Hopf orders can then be compared with those of Greither-Childs (1998) and Childs-Smith III (2005).

**Duc Huynh.**

*Enumerating $k$-isomorphism classes of totally ramified degree-$p$ extensions of the local field $k((\pi))$.* 50 Minutes

Abstract: Let $K$ be a local field of characteristic $p$ with finite residue field $k$. Let $K_s$ be be a separable closure of $K$. For $\lambda \in \mathbb{Q}$, let $\mathcal{E}_\lambda$ be the set of all totally ramified field extensions in $K_s$ of degree $p$ over $K$ with ramification break $\lambda$. We will enumerate the $k$-isomorphism classes of $\mathcal{E}_\lambda$.

**Kevin Keating.**

*Parametrizing Extensions of Local Fields.* 50 Minutes

Abstract: Let $k$ be an algebraically closed field of characteristic $p$, and set $F = k((t))$. In their paper "Configuration spaces for wildly ramified covers", Fried and Mezard describe a rather simple affine variety which parametrizes $F$-isomorphism classes of finite separable extensions $E/F$ with specified degree and ramification data. In this expository talk I will describe the construction of Fried and Mezard and consider whether and how it might be useful in studying local field extensions.

**Alan Koch.**

*Hopf Galois Structures on Modular Extensions.* 25 Minutes

Abstract: Let $K$ be an imperfect field of characteristic $p > 0$. We introduce a class of monogenic local $K$-Hopf algebras with local linear duals. Let $L$ be a primitive purely inseparable extension of $K$ of degree

$p^n$ and let $H$ one of our $K$-Hopf algebras of the same dimension. We show that $L$ can be viewed as an $H$-Galois object using several different coactions. From this it follows that there are many, in some cases infinitely many, $K$-Hopf algebras $H$ such that $L/K$ is $H$-Galois. The Hopf Galois action is made explicit for a subclass of our constructed Hopf algebras. Finally, we generalize our results beyond primitive extensions, providing many Hopf Galois structures for every modular extension $L/K$.

*Hopf Galois Scaffolds on Purely Inseparable Extensions.* 50 Minutes

Abstract: Let $p$ be prime. Let $L/K$ be a finite purely inseparable extension of local fields, $[L : K] = p^n,\ n \geq 2$. From our previous talk, we know that $L/K$ is Hopf Galois for numerous Hopf algebras $H$. For a certain collection of such $H$ we construct "Hopf Galois scaffolds", $A$-scaffolds in the sense of Byott-Childs-Elder where $A$ is a $K$-Hopf algebra, obtaining an analogue to the Normal Basis Theorem for $L/K$. The existence of a scaffold structure is independent of the chosen action of $H$ on $L$. We apply the theory of scaffolds to describe when the fractional ideals of $L$ are free over their associated orders in $H$.

**Dave Roberts.**

*A database of number fields.* 25 Minutes

Abstract: For many years, John Jones of Arizona State University and I have been building up an online database of number fields at `http://hobbes.la.asu.edu/NFDB/`. The database is centered on user queries which return all fields on the database with the requested properties. Its main feature is that it certifies completeness of the returned list whenever possible. Also of particular interest for this workshop is that the database pays close attention to p-adic ramification. I will give an informal presentation explaining in practical terms how one uses the database. [A recent preprint describing the database is available at `http://facultypages.morris.umn.edu/~roberts/research/publications.html`]

*Primitive extensions of $\mathbb{Q}_p$ and trinomial-Eisenstein polynomials.* 25 Minutes

Abstract: Primitive totally ramified extensions of $\mathbb{Q}_p$ of a given degree $p^k$ and a given discriminant $p^c$ are hard to distinguish from each other by standard invariants. I discuss the problem of explicitly distinguishing such fields. Some of these extensions can be given by Eisenstein polynomials which have only three terms while others cannot. It seems that this distinction is meaningful and can be taken as the starting point of a full classification. I do not fully understand this situation and am hoping that a workshop participant will have some helpful ideas.

**Lena Sundukova.**

*Local Galois module structure in prime characteristic and Galois scaffolds.* 50 Minutes

Abstract: Let $L/K$ be a finite totally ramified Galois extension of local fields. A set of conditions imposed on $L/K$, called Galois scaffold, help describe its integral Galois module structure as well as the structure of fractional ideals of $L$ over their respective associated orders. The results are closely related to the ramification numbers of $L/K$.

**Paul Truman.**

*Canonical Nonclassical Hopf-Galois Module Structure of Nonabelian Galois Extensions.* 50 Minutes

Abstract: Let $L/K$ be a finite Galois extension of local or global fields in characteristic 0 or $p$ with nonabelian Galois group $G$. By the theorem of Greither and Pareigis, $L/K$ has a canonical nonclassical Hopf-Galois structure whose Hopf algebra $H$ corresponds to the left regular embedding of $G$ into $\mathrm{Perm}(G)$. We compare the structure of $\mathfrak{O}_L$ as a module over its associated order in $K[G]$ and over its associated order in $H$.

*Hopf-Galois Structures on Quaternionic Extensions and some examples of Martinet's.* 50 Minutes

Abstract: Jacques Martinet famously discovered examples of tamely ramified Quaternionic extensions of the rational numbers whose rings of algebraic integers are locally, but not globally, free over the integral group ring. More recently, Nigel Byott discovered examples of wildly ramified Galois extensions of $p$-adic fields in which $\mathfrak{O}_L$ is not free over its associated order in $K[G]$ but is free over its associated order in some Hopf algebra giving a nonclassical Hopf-Galois structure on the extension. Motivated by these examples, we ask whether the use of nonclassical Hopf-Galois structures might give a more satisfactory description of the rings of algebraic integers in Martinet's tame Quaternionic extensions.

**Rob Underwood.**

*Duals of Hopf Orders in Characteristic p.* 25 Minutes

Abstract: Let $p$ be a prime number, let $n \geq 1$ be an integer and let $\mathbb{F}_q$ denote the field with $q = p^n$ elements. Let $t$ be an indeterminate and let $R = \mathbb{F}_q[[t]]$ and $K = \mathrm{Frac}(R) = \mathbb{F}_q((t))$. Let $C_p^2$ denote the elementary abelian group of order $p^2$ with $C_p^2 = \{\sigma^a\tau^b\}$, $0 \leq a, b \leq p-1$. Let $i, j \geq 0$ be integers and let $\mu \in K$ be so that $\mathrm{ord}(\mu) \geq -i + (j/p)$. There exists an $R$-Hopf order in $KC_p^2$ of the form

$$H = H_{i,j,\mu} = R\left[\frac{\sigma - 1}{t^i}, \frac{\sigma^{[-\mu]}\tau - 1}{t^j}\right],$$

where

$$\sigma^{[-\mu]} = \sum_{m=0}^{p-1} \binom{-\mu}{m} (\sigma - 1)^m.$$

In this talk we compute the linear dual $H^* \subseteq (KC_p^2)^*$.

## 1. Additional Information

**Design of conference.** Talks are either 25 or 50 minutes. Quite often, people give two talks.

One of the features that everyone has enjoyed in previous conferences has been the inclusion of guiding principles, half-backed ideas, crazy conjectures. Please come with some to share.

**Lecture space.** The talks will be in Durham Science Center Room 254. There is a computer and projector connected to the Internet, along with a side blackboard. So, if you would like, you can use both at the same time. There is also an ELMO, a document camera for projecting images from paper. And if you would like to use transparencies, it is possible with an advance request.

**Coffee.** There is a Starbucks Café in the Library next door to Durham Science Center. But we will also have a coffee machine in the room, pitcher of water/glasses along with fruit (apples, oranges, bananas) and bagels with cream cheese.

**Wi-Fi.** On campus there is Wireless available. Visit `http://is.unomaha.edu/systems/wireless.php`.