SCHEDULE: HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 23–27, 2016

Monday.

9:00am *Welcome*, Dave Boocker (Dean, College of Arts & Sciences)

9:30am Koch: *Dieudonné modules, Part I: a study of primitively generated Hopf algebras.* 60 min.

11:00am Childs: *Applications of nilpotent algebras to Hopf Galois structures.* 60 min.

1:30pm Keating: *What is a refined ramification break?* 30 min.

2:30pm Elder: *Examples: Refined ramification under a Galois scaffold.* 30 min.

Tuesday.

9:30am Koch: *Dieudonné module theory, Part II: the classical theory.* 60 min.

11:00am Kohl: *Regular and semi-regular permutation groups and their centralizers and normalizers.* 60 min.

1:30pm Truman: *Commuting Hopf-Galois structures on a separable extension.* 60 min.

Evening One of only 4 remaining great Omaha Steakhouses, Johnny's Cafe `http://www.johnnyscafe.com/`, as featured in the 2002 Alexander Payne movie, *About Schmidt*.

Wednesday.

9:30am Koch: *Dieudonné module theory, Part III: applications.* 60 min.

11:00am Kohl: *Hopf-Galois structures on degree mp extensions.* 60 min.

1:30pm Greither: *Module theory and Cartan matrices: a result of Schneider and its context.* 60 min.

Thursday.

9:30am Keating: *Trace, Norm, etc.* 60 min.

11:00am Greither: *Generating the root of the codifferent by values of elliptic functions.* 60 min.

1:30pm Childs: *Hopf Galois structures for totally ramified p-elementary abelian Galois extensions.* 30 min.

2:30pm Kohl: *Regular and semi-reg. permutation groups and their centralizers and normalizers II.* 30 min.

3:30pm Truman: *Commutative Hopf-Galois module structure of tame extensions.* 30 min.

Friday.

9:30am Byott: *Some counting problems for Hopf-Galois structures.* 60 min.

11:00am Underwood: *The structure of Hopf algebras acting on Galois extensions.* 60 min.

Afternoon Group bike ride on the Keystone Trail.

Evening Pool party at the Elder-berry Residence: 5624 Leavenworth St.

Abstracts

Nigel Byott.

Some counting problems for Hopf-Galois structures. 60 minutes

Abstract: Let N/K be a finite, Galois field extension with Galois group Γ. Greither and Pareigis (1987) showed that the Hopf-Galois structures on N/K correspond to certain subgroups G of order $|\Gamma|$ in $\mathrm{Perm}(\Gamma)$. This raises the problem of counting all Hopf-Galois structures on N/K, or all those Hopf-Galois structures where G has a given isomorphism type.

I will describe joint work with my PhD student Ali Bilal which counts the Hopf-Galois structures on a cyclic extension of squarefree order (by type and in total), and outline our ongoing attempts to extend this to arbitrary Galois extensions of squarefree degree. I will also propose a possible strategy to count all Hopf-Galois structures on a cyclic extension of arbitrary degree.

Lindsay Childs.

Applications of nilpotent algebras to Hopf Galois structures. 60 minutes

Abstract: Let L/K be a Galois extension of fields with Galois group G, an elementary abelian p-group of order p^n, p an odd prime. For $n > 1$, L/K has non-classical Hopf Galois structures of type G, which have been studied by looking at regular subgroups of $\mathrm{Hol}(G)$ that are isomorphic to G. Think of G as an additive group $(G, +)$. This talk will exploit the connection, introduced by Caranti, Della Volta and Sala in 2006, between regular subgroups of $\mathrm{Hol}(G)$ and associative, commutative nilpotent algebra structures A on $(G, +)$. We briefly review three known consequences of this connection for Hopf Galois structures. Then we describe how the connection helps understand the lack of surjectivity of the Galois correspondence from subHopf algebras to subfields given by the Chase-Sweedler Fundamental Theorem of Galois Theory for a Hopf Galois structure on L/K.

Hopf Galois structures for totally ramified p-elementary abelian Galois extensions. 30 minutes

Abstract: Among the many non-classical Hopf Galois structures on a Galois extension L/K of fields with Galois group G, an elementary abelian p-group of order p^n, p an odd prime, we will introduce a family that might be particularly suitable for the case where L/K is a totally ramified extension of local fields of residue characteristic p with a maximal number of distinct ramification numbers.

Griff Elder.

Examples: Refined ramification under a Galois scaffold. 60 minutes

Abstract: Refined ramification originated with an attempt to determine Galois module structure in $C_p \times C_p$-extensions with one ramification break. After about ten years, these are still the only extensions for which a

fully satisfactory theory exists. We need more examples, and that is the purpose of this talk. So that we can see through the myriad technical details that cloud the picture for $C_p \times C_p \times C_p$-extensions and beyond, I will restrict myself to extensions with a Galois scaffold. For this restricted class of extensions, I will establish some of the features that I believe are desirable in a theory of refined ramification.

Cornelius Greither.

Generating the root of the codifferent by values of elliptic functions. 60 minutes

This is a preliminary report on work in slow progress.

Abstract: It is well known that the square root $A_{L/K}$ of the codifferent in a weakly ramified G-Galois extension of p-adic fields is a free $O_K[G]$-module. We will also assume that L/K is totally ramified. Then G is elementary p-abelian. The case $K = \mathbb{Q}_p$ is nice and simple: here G is at most cyclic of order p, L is "essentially" the degree p subfield of $\mathbb{Q}(\zeta_{p^2})$, and Erez gave a generator of $A_{L/K}$ in terms of ζ_{p^2}. More general constructions were given by Pickett-Vinatier and by the author. They are still very cyclotomic in spirit, using Kummer extensions. New work of Pickett-Thomas involves formal groups. So it is tempting to look around for a construction that combines both aspects: globality, and the use of algebraic groups. We try to do this in a very modest case: K is unramified quadratic over $\mathbb{Q}_p$, and G is bi-cyclic of order p^2 (which is the largest possible for such k). We pick the elliptic curve $E : y^2 = x^3 + x$ defined over $\mathbb{Q}$, and we restrict to $p \equiv 3 \pmod 4$, so $K = \mathbb{Q}(i)$. Note that E has complex multiplication with $\mathbb{Z}[i]$. We do find a local generator for $A_{L/K}$ with global origin: it comes from division values of an appropriate elliptic function on E. It is an important feature of our approach that the extension L/K is the completion of an abelian extension of $\mathbb{Q}(i)$ which has the same Galois group G and is unramified outside p. So far we haven't succeeded in finding a global generator. Indeed, the generators which we exhibit fail to generate the root of the global codifferent at certain primes (of course finite in number), and we do not yet understand the nature and origin of those bad primes.

Module theory and Cartan matrices: a result of Schneider and its context. 60 minutes

Abstract: We will begin by explaining the background, made up by module theory and a little K-theory. Then we'll explain the Cartan matrix and the Cartan-Brauer triangle in detail and try to elucidate these concepts by simple examples. Then we will state Schneider's result which says that for liftable Hopf algebras the Cartan matrix is invertible, and explain the important consequence in Hopf Galois theory: two projective H-modules are isomorphic as soon as they become isomorphic over K. Here H is a finite Hopf algebra over the complete DVR R, which has characteristic 0 and has p in its radical (so to speak, H is already a lifted object); K is the quotient field of R. We will probably not be able to give a complete proof of Schneider's

result on the Cartan matrix, and will make do with a very brief sketch. We think that the important thing is to get a feel for the algebraic concepts, and to see the relevance of Schneider's result for Hopf Galois theory.

Kevin Keating.

What is a refined ramification break? 30 minutes

Abstract: Let L/K be a finite totally ramified Galois extension of degree $n = ap^\nu$. In most cases one expects L/K to have ν distinct positive (lower) ramification breaks. There have been attempts to supply the "missing" data in the cases where there are fewer than ν breaks. Some of these approaches are based on Galois module theory, as in the work of Byott-Elder. In this talk I will survey several possible definitions of refined ramification breaks which are based on Galois module theory. At the end of the talk I will survey the audience for their opinions on which is the correct definition to use.

Trace, Norm, etc. 60 minutes

Abstract: Let K be a local field with valuation v_K. Let K^{sep} be a separable closure of K, and let L/K be a finite totally ramified subextension of K^{sep}/K of degree n. Let $\sigma_1, \ldots, \sigma_n$ denote the K-embeddings of L into K^{sep}. For $1 \le i \le n$ let $s_i(X_1, \ldots, X_n)$ denote the ith elementary symmetric polynomial in n variables, and for $\alpha \in L$ set $S_i(\alpha) = s_i(\sigma_1(\alpha), \ldots, \sigma_n(\alpha))$. In this talk we consider the problem of finding a lower bound for $v_K(S_i(\alpha))$ in terms of $v_L(\alpha)$. The solution seems to depend on the indices of inseparability of the extension L/K.

Alan Koch.

Dieudonné modules, Part I: a study of primitively generated Hopf algebras. 60 minutes

Abstract: Let R be an $\mathbb{F}_p$-algebra, and let H be a primitively generated R-Hopf algebra. Then the group $P(H)$ of primitive elements of H can be viewed as a module over the non-commutative polynomial ring $R[F]$ with $Fa = a^pF$ for all $a \in R$. We illustrate how $H \mapsto P(H)$ is a one-to-one correspondence between primitively generated R-Hopf algebras and finite-length $R[F]$-modules. We give applications of this "Dieudonné" correspondence, showing how it facilitates the study of endomorphisms, extensions, forms, and Hopf orders for certain choices of R, for example when R is a discrete valuation ring.

Dieudonné module theory, Part II: the classical theory. 60 minutes

Abstract: Let k be a perfect field, $\operatorname{char} k = p > 2$. Classic Dieudonné module theory describes finite commutative connected unipotent group schemes of p-power rank over k, associating to each such group scheme a module over a certain ring E called a Dieudonné module. Since any such group scheme is represented by a finite commutative cocommutative local-local k-Hopf algebra, the correspondence above can be reinterpreted as one between these Hopf algebras and Dieudonné modules. The literature tends to focus on the group scheme interpretation of the theory; here, we will focus on the Hopf algebras instead. Starting with the construction of the ring of Witt vectors, we will construct the Dieudonné ring E and explain the correspondence. We will give elementary properties, discuss duality, and work through many examples to illustrate explicitly how the correspondence works.

Dieudonné module theory, Part III: applications. 60 minutes

Abstract: Let k be a perfect field, $\operatorname{char} k = p > 2$. Continuing from Tuesday's talk, and in the spirit of Monday's, we will show how certain questions about Hopf algebras can be solved using Dieudonné modules. In particular, we will show how Dieudonné modules allow for a classification of monogenic k-Hopf algebras (finite, abelian, local-local), k-Hopf algebras with two generators (as a k-algebra), and height one k-Hopf algebras. We conclude by discussing how Dieudonné modules can be extended to all p-power rank k-Hopf algebras, and talk about "Dieudonné-type" theories over discrete valuation rings.

Tim Kohl.

Hopf-Galois structures on degree mp extensions. 60 minutes

Abstract: Let L/K be a Galois extension with $\Gamma = Gal(L/K)$ where $[L : K] = mp$ for p a prime where $gcd(p, m) = 1$. Assume also that p and m are such that any group of order mp has a unique Sylow p-subgroup and that p does not divide the order of the automorphism group of any group of order m. Under these conditions, regular subgroups $N \leq B$ that give rise to Hopf-Galois structures on L/K are contained in $Norm_B(\mathcal{P})$ where $\mathcal{P}$ is the Sylow p-subgroup of $\lambda(\Gamma) \leq B$. We include specific applications of this result to enumerate the Hopf-Galois structures on a variety of different families of extensions of degree mp.

Regular and semi-regular permutation groups and their centralizers and normalizers. 60 minutes

Abstract: The Hopf-Galois structures on a separable field extension correspond to regular subgroups N of an ambient symmetric group $B = Perm(X) \cong S_{|X|}$ where X is either the elements of the Galois group or the cosets of the Galois group of the normal closure. We give an overview of the definitions and consequences of regularity and semi-regularity. Going further we consider centralizers and normalizers of these, again with a view towards the classification of Hopf-Galois structures. We shall see the interplay between general group theoretic properties as well as the combinatorial aspects inherent from the action(s) of these groups on the underlying set X.

Regular and semi-regular permutation groups and their centralizers and normalizers II. 30 minutes

Abstract: The Hopf-Galois structures on a Galois extension K/k where $\Gamma = Gal(K/k)$ are in one-to-one correspondence with the regular subgroups $N \leq B = Perm(\Gamma)$ that are normalize by $\lambda(\Gamma)$. More broadly, one is enumerating the set(s) $R(\Gamma, [M])$ which consist of all those regular subgroups normalized by $\lambda(\Gamma)$ that are isomorphic to the abstract group M, where M runs over all the isomorphism classes of groups of order $n = [K : k]$. For X a finite set, where $|X| = n$, we consider representative regular subgroups $\Gamma_1, \ldots, \Gamma_r$ of $B = Perm(X)$ where r is the number of abstract groups of order n. The regular subgroups of B fall into exactly as many conjugacy classes as isomorphism classes, so that any regular subgroup of B is conjugate to exactly one of these $\{\Gamma_j\}$. For these we consider the relationship between the sets $S(\Gamma_j, [\Gamma_j])$, consisting of those regular subgroups of $Norm_B(\Gamma_j)$ that are isomorphic to Γ_i, and $R(\Gamma_i, [\Gamma_j])$ which is the set of those regular subgroups of B that are isomorphic to Γ_j and normalized by Γ_i. In particular we give a re-capitulation of various enumerative schemes that rely on the correspondence between regular subgroups and regular embeddings. We also consider the 'isomorphic' equals 'conjugate' idea and how it can be used in the enumeration of Hopf-Galois structures.

Paul Truman.

Commuting Hopf-Galois structures on a separable extension. 60 minutes

Abstract: Let L/K be a finite separable extension of local or global fields in any characteristic, let H, H' be Hopf algebras giving Hopf-Galois structures on L/K, and suppose that the actions of H, H' on L commute. We show that a fractional ideal of L is free over its associated order in H if and only if it is free over its associated order in H', and investigate some properties that these associated orders share. In the case that L/K is a totally ramified extension of local fields with residue characteristic p and p-power degree, we investigate conditions under which the existence of an H-scaffold implies the existence of an H'-scaffold.

Commutative Hopf-Galois module structure of tame extensions. 30 minutes

Abstract: We use the concept of induced Hopf-Galois structures to show that if L/K is a Galois extension of p-adic fields which is at most tamely ramified and H is a commutative Hopf algebra giving a Hopf-Galois structure on the extension then O_L is free over its associated order in H. We also study the case in which L/K is a non-normal extension of p-adic fields, and the case in which L/K is a Galois extension of number fields.

Rob Underwood.

The structure of Hopf algebras acting on Galois extensions. 60 minutes

Abstract: Let $K/\mathbb{Q}$ be a Galois extension with group G. Then by Greither-Pareigis theory, there is a one-to-one correspondence between Hopf-Galois structures on $K/\mathbb{Q}$ and regular subgroups of Perm(G) which are normalized by $\lambda(G)$, where λ is the left regular representation of G in $\mathrm{Perm}(G)$. All of the Hopf algebras thus constructed are finite dimensional algebras over $\mathbb{Q}$. In this talk, we discuss the Wedderburn-Malcev decompositions of these Hopf algebras.

1. Additional Information

Design of conference. Talks are either 30 or 60 minutes.

In previous conferences, one of the features that everyone has enjoyed has been the inclusion of guiding principles, half-backed ideas, crazy conjectures. Please come with some to share.

Lecture space. The talks will be in Durham Science Center Room 254. There is a computer and projector connected to the Internet, along with a regular blackboard. So, if you would like, you can use both at the same time. There is also an ELMO, a document camera for projecting images from paper. And if you would like to use transparencies, it is possible with an advance request.

Coffee, snacks & lunch. There is a Starbucks in the Library next door to Durham Science Center that is open 10:30-2:00 each day. But we will also have a coffee machine in the room, pitcher of water/glasses along with fruit (apples, oranges, bananas) and bagels with cream cheese.

Lunch. The Food court in the Milo Bail Student center will be open from 7:30 till 2:00pm each day (Mexican, Asian and Italian food, as well as burgers, subway sandwiches, etc). Other options include Fuddruckers, Noodles & Company, Vietnamese-Thai Restaurant all slightly west of campus on 72nd St.

Wi-Fi. There is Wireless available, including *eduroam*. Visit `http://www.unomaha.edu/information-services/networks-and-connectivity.php`