SCHEDULE: HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 21–25, 2018

Monday.

9:00am *Welcome*, Dave Boocker (Dean, College of Arts & Sciences)

9:30am Keating: *Bondarko's work on local additive Galois module theory: What he did.* 60 minutes

11:00am Koch: *Greither-Pareigis theory, through the lens of algebraic geometry.* 60 minutes

1:30pm Kohl: *Characteristic Subgroup Lattices and Hopf-Galois Structures.* 60 minutes

3:00pm Truman: *Hopf-Galois module structure of weakly ramified extensions.* 60 minutes

Tuesday.

9:30am Keating: *Bondarko's work on local additive Galois module theory: How he did it.* 60 minutes

11:00am Childs: *On the Galois correspondence for Hopf Galois structures, I.* 60 minutes

1:30pm Koch: *Isomorphisms within Hopf-Galois structures on separable field extensions.* 60 minutes

Evening One of only 4 remaining great Omaha Steakhouses, Johnny's Cafe `http://www.johnnyscafe.com/`, as featured in the 2002 Alexander Payne movie, *About Schmidt*.

Wednesday.

9:30am Keating: *Bondarko's work on local additive Galois module theory: What to do with it.* 60 minutes

11:00am Underwood: *Hopf-Galois Structures and a Characterization of Dihedral Extensions.* 60 minutes

1:30pm Childs: *On the Galois correspondence for Hopf Galois structures, II.* 30 minutes

Evening Informal dinner: potroast, veg chili (both from slow cooker), mash potatoes and green beans.

Thursday.

9:30am Kohl: *Multiple Holomorphs and Hopf-Galois Structures.* 60 minutes

11:00am Truman: *Extensions of classical Hopf-Galois structures.* 60 minutes

1:30pm Taylor: *Hopf-Galois module structure of a class of tame Quaternionic fields.* 60 minutes

3:00pm Horner: *Adventures in S^3.* 30 minutes

Friday.

9:30am Elder: *Sharp lower bounds on ramification breaks in extensions of degree p^3.* 60 minutes

Afternoon Group hike at Hitchcock Park, Iowa.

Evening Pool party at the Elder-berry Residence: 5624 Leavenworth St.

Abstracts

Lindsay Childs, University of Albany.

On the Galois correspondence for Hopf Galois structures, I. 60 minutes

Abstract: Let L/K be a Galois extension of fields with Galois group Γ, and suppose L/K is also an H-Hopf Galois extension of type G. Using the connection between Hopf Galois structures and skew left braces, we introduce a method to quantify the failure of surjectivity of the Galois correspondence from subHopf algebras of H to intermediate subfields of L/K, given by the Fundamental Theorem of Hopf Galois Theory. Suppose $L \otimes_K H = LN$ where $N \cong (G, \star)$ (so the Hopf Galois structure has type G). Then there exists a skew left brace $(G, \star, \circ)$ where $(G, \circ) \cong \Gamma$. We show that there is a bijective correspondence between intermediate fields E between K and L and certain sub-skew left braces of G, which we call the $\circ$-stable subgroups of $(G, \star)$. Counting the $\circ$-stable subgroups and comparing that number with the number of subgroups of $\Gamma \cong (G, \circ)$ describes how far the Galois correspondence for the H-Hopf Galois structure is from being surjective. The method is illustrated by a variety of examples.

On the Galois correspondence for Hopf Galois structures, II. 30 minutes

Abstract: We apply the left skew brace approach to understanding the image of the Galois correspondence for a Hopf Galois extension in the setting where the Hopf Galois extension corresponds to a pair of fixed point free homomorphisms from the Galois group Γ to the type group G.

Griff Elder, University of Nebraska at Omaha.

Sharp lower bounds on ramification breaks in extensions of degree p^3 60 minutes.

Abstract: Let L/K be a totally ramified, Galois extension of degree p^3 where K is a characteristic p local field with perfect residue field. If the Galois group is abelian, a classification of all possible ramification breaks is available. In all cases, there is a description of the largest ramification break in terms of smaller ramification breaks. This description has three features: (1) There is a lower bound. (2) All integers above this lower bound that satisfy certain conguences are possible. (3) The lower bound doesn't satisfy the certain congruences. In this talk, I will consider this classification for nonabelian Galois groups. The same features arise. We will dwell on the most interesting feature, the lower bound.

Kevin Keating, University of Florida.

Bondarko's work on local additive Galois module theory: What he did. 60 minutes

Abstract: In this talk I will survey the work of Bondarko on additive Galois module theory. The main question which he considers is the "local Leopoldt problem" for a finite totally ramified extension L/K of

local fields with Galois group G. This problem asks whether the ring of integers of L (or, more generally, a power of the maximal ideal of L) is free over its associated order in $K[G]$.

Bondarko's work on local additive Galois module theory: How he did it. 60 minutes

Abstract: In this talk I hope to give some idea of the methods used by Bondarko to prove his results on additive Galois module theory, focusing of course on the simplest cases.

Bondarko's work on local additive Galois module theory: What to do with it. 60 minutes

Abstract: In this talk I will consider some possible ways to apply and extend Bondarko's work, possibly including some Crazy Ideas.

Alan Koch, Agnes Scott College.

Greither-Pareigis theory, through the lens of algebraic geometry. 60 minutes

Abstract: We give a group scheme-theoretic description of the Hopf Galois theory over separable field extensions. Our motivation is twofold. First, we wish to simplify certain isomorphism problems addressed in a recent paper by Koch, Kohl, Truman, and Underwood, namely: under what conditions do two Hopf-Galois structures on a separable field extension L/K have underlying Hopf algebras isomorphic to each other as K-algebras? Second, we wish to determine if this allows for an adaptation of Greither-Pareigis theory to purely inseparable field extensions; the existence of such a theory is a question posed by Childs in 2013.

Isomorphisms within Hopf-Galois structures on separable field extensions. 60 minutes

Abstract: Let L/K be a separable extension with Galois closure E, and let H_1 and H_2 be two K-Hopf algebras which provide Hopf-Galois structures on L/K. We will give a criterion for H_1 and H_2 to be isomorphic as K-Hopf algebras; moreover we will describe when $H_1 \otimes_K F \cong H_2 \otimes_K F$ as F-Hopf algebras for some field $K \subseteq F \subseteq E$. In the case $H_1,\ H_2$ commutative we will also describe when$H_1 \cong H_2$ as K-algebras and $H_1 \otimes_K F \cong H_2 \otimes_K F$ as F-algebras. Examples will be given in certain cases where L/K is Galois, group G, notably the cases where G is dihedral; elementary abelian of prime power degree; and cyclic of prime power degree.

Tim Kohl, Boston University.

Characteristic Subgroup Lattices and Hopf-Galois Structures. 60 minutes

Abstract: For K/k a Galois extension with group G where $[K : k] = |G| = n$, a Hopf-Galois structure arises due to the action of a Hopf algebra of the form $H = (K[N])^{\lambda(G)}$ where $N \leq B = Perm(G)$ is a regular permutation group, normalized by $\lambda(G) \leq B$, the left regular representation. Such an 'N' must belong to some isomorphism class $[M]$ of groups, necessarily of order n, but need not be isomorphic to G itself. The totality of all such groups is denoted $R(G,[M])$. For such an N, there must arise an injective correspondence between the characterstic sugroups of N and the subgroups of G. We utilize this to infer that $R(G,[M])$ must be empty for quite a number of such pairings, for a given order n.

Multiple Holomorphs and Hopf-Galois Structures. 60 minutes

Abstract: For an abstract group G, the holomorph $Hol(G)$ is the normalizer in $B = Perm(G)$ of the left regular representation $\lambda(G) \leq B$. The multiple holomorph $NHol(G)$ is the normalizer in B of $Hol(G)$, and the quotient group $T(G) = NHol(G)/Hol(G)$ exactly parametrizes the set of those other regular subgroups of $N \leq B$, which are isomorphic to G and are such that $Norm_B(N) = Norm_B(\lambda(G)) = Hol(G)$. For K/k a Galois extension with group G where $|G| = [K : k] = n$, a Hopf-Galois structure arises due to the action of a Hopf algebra of the form $H = (K[N])^{\lambda(G)}$ where $N \leq B = Perm(G)$ is a regular permutation group, normalized by $\lambda(G) \leq B$. Such an 'N' need not be isomorphic to G, but belongs to some isomorphism class $[M]$ of groups, necessarily of order n. If $R(G,[M])$ is the collection of all such N, we show that $|T(M)|$ divides $|R(G,[M])|$. We also show some preliminary results on how to utilize two different group actions on $R(G,[M])$ to give estimates/bounds on $|R(G,[M])|$ overall, in terms of $|Aut(G)|$ and $|T(M)|$.

Stuart Taylor, Keele University.

Hopf-Galois module structure of a class of tame Quaternionic fields. 60 minutes

Abstract: Galois extensions with Galois group isomorphic to the Quaternion group of order 8 have been important in the history of Galois module structure. Martinet gave examples of tame Quaternionic extensions of $\mathbb{Q}$ with no normal integral basis. We study the Hopf-Galois module structure of some of these extensions.

Paul Truman, Keele University.

Hopf-Galois module structure of weakly ramified extensions. 60 minutes

Abstract: A finite Galois extension of local fields L/K with Galois group G is said to be weakly ramified if the second ramification group is trivial. A result of Johnston states that if L/K is weakly ramified then a fractional ideal $\mathfrak{P}_L^n$ of L is free over $\mathfrak{O}_K[G]$ if and only if $n \equiv 1 \pmod{|G_1|}$, and $\mathfrak{O}_L$ is free over its associated order in $K[G]$. We investigate generalizations of this result to nonclassical Hopf-Galois structures.

Extensions of classical Hopf-Galois structures. 60 minutes

Abstract: In a recent preprint with Koch, Kohl and Underwood we investigated the following question: given a finite Galois extension of fields L/K and a subextension F/K, does a Hopf-Galois structure on L/K yield Hopf-Galois structures on L/F and F/K? In this talk we turn the situation around and ask whether Hopf-Galois structures on L/F and F/K can be used to induce a Hopf-Galois structure on L/K. We restrict

ourselves to the case in which F/K is Galois and the Hopf-Galois structures on L/F and F/K are both classical.

Rob Underwood, Auburn University at Montgomery.

Hopf-Galois Structures and a Characterization of Dihedral Extensions. 60 minutes

Abstract: Let L/K be a Galois extension with non-abelian group G. Then L/K admits both a classical and canonical non-classical Hopf-Galois structure via the Hopf algebras $K[G]$ and H_λ, respectively. By a theorem of C. Greither, $K[G] \cong H_\lambda$ as K-algebras. In this talk we apply Greither's result to the case $K = \mathbb{Q}$, $G = D_3$ to yield a characterization of Galois extensions with group D_3. In the case $G = D_4$, Greither's theorem can be applied to a result of A. Ledet.

This is joint work with A. Koch, T. Kohl, and P.J. Truman.

1. Additional Information

Design of conference. Talks are either 30 or 60 minutes.

In previous conferences, one of the features that everyone has enjoyed has been the inclusion of guiding principles, half-backed ideas, crazy conjectures. Please come with some to share.

Lecture space. The talks will be in Durham Science Center Room 254. There is a computer and projector connected to the Internet, along with a regular blackboard. So, if you would like, you can use both at the same time. There is also an ELMO, a document camera for projecting images from paper. And if you would like to use transparencies, it is possible with an advance request.

Coffee, snacks & lunch. There is a Starbucks in the Library next door to Durham Science Center that is open 10:30-2:00 each day. But we will also have a coffee machine in the room, pitcher of water/glasses along with fruit (apples, oranges, bananas) and bagels with cream cheese.

Lunch. The Food court in the Milo Bail Student center will be open from 7:30 till 2:00pm each day (Mexican, Asian and Italian food, as well as burgers, subway sandwiches, etc). Other options include Fuddruckers, Noodles & Company, Vietnamese-Thai Restaurant all slightly west of campus on 72nd St.

Wi-Fi. There is Wireless available, including *eduroam*. Visit `http://www.unomaha.edu/information-services/networks-and-connectivity.php`