

BRACES AND HOPF-GALOIS THEORY
KEELE UNIVERSITY, 19 JUNE 2019

SCHEDULE

- 9:00am** Zenouz: *The Yang-Baxter equation and Hopf-Galois theory via skew braces*
- 10:00am** Coffee
- 10:30am** Byott: *Hopf-Galois structures on Galois extensions of squarefree degree, and skew braces of squarefree order*
- 11.30am** Samways: *Enumeration of Hopf-Galois structures on cyclic field extensions*
- 12:00pm** Lunch
- 1:30pm** Brzezinski: *Introducing trusses*
- 2:30pm** Taylor: *Isomorphism problems for Hopf-Galois structures on quaternion extensions*
- 3:00pm** Truman: *Opposite Hopf-Galois structures and opposite braces*
- 3:30pm** Tea

ABSTRACTS

Thomas Brzezinski (Swansea University)

Introducing trusses (60 Minutes)

Abstract: A truss is an algebraic system consisting of a set, a ternary operation (that makes it into an Abelian heap) and an associative binary operation that distributes over the ternary one. By specifying an element, the binary-ternary truss distributive law can be converted into a modified binary-binary distributive law that, as special cases, includes the usual ring-type law or a brace-type distributive law. In this talk we give a short introduction to trusses, explain their connection with braces, and point-out some ways in which they differ from rings.

Nigel Byott (University of Exeter)

Hopf-Galois structures on Galois extensions of squarefree degree, and skew braces of squarefree order (60 Minutes)

Abstract: Given finite groups Γ , G of the same order n , the problem of counting Hopf-Galois structures of type G on a Galois extension of fields with Galois group Γ is closely related, but not equivalent, to the problem of counting left skew braces (up to isomorphism) with additive group G and multiplicative group Γ . I will outline the connection between these two problems, and describe joint work with Ali Alabdali which solves both problems in the case that n is squarefree.

George Samways (University of Exeter)

Enumeration of Hopf-Galois structures on cyclic field extensions (30 Minutes)

Abstract: Let L/K be a Galois extension of fields with cyclic Galois group of order n . We seek to enumerate the Hopf-Galois structures on such extensions, building on earlier work which placed certain restrictions on the order of n . By Greither and Pareigis, every Hopf algebra giving rise to a Hopf-Galois structure on L/K may be associated with some abstract group G of order n , called the type of the structure. In particular, these groups G must be supersolvable, and every p -Sylow subgroup of G is cyclic for p an odd prime. When the 2-Sylow subgroup of G is also cyclic, we obtain results on the number of structures in terms of the type G .

Stuart Taylor (Keele University)

Isomorphism problems for Hopf-Galois structures on quaternion extensions (30 Minutes)

Abstract: Let L/K be a Galois extensions of fields whose Galois group G is isomorphic to the quaternion group of order 8. By the Greither-Pareigis classification, the Hopf-Galois structures on L/K correspond to certain regular subgroups of $\text{Perm}(G)$ of order 8. We give explicit descriptions of these regular subgroups, and determine which of the Hopf algebras that arise are isomorphic as K -Hopf algebras or as K -algebras.

Paul Truman (Keele University)

Opposite Hopf-Galois structures and opposite braces (30 Minutes)

Abstract: Let L/K be a Galois extension of fields. If H is a Hopf algebra giving a Hopf-Galois structure on L/K then there is a corresponding “opposite” Hopf-Galois structure, with Hopf algebra H' , say. We study the relationships between H and H' . In particular, we prove that H and H' are isomorphic as K -algebras, and give evidence in support of a conjecture that they are never isomorphic as K -Hopf algebras. Translating into the language of braces, we formulate a notion of an “opposite” brace, and study properties of this construction.

Kayvan Nejabati Zenouz (Oxford Brookes University)

The Yang-Baxter Equation and Hopf-Galois Theory via Skew Braces (60 Minutes)

Abstract: The Yang-Baxter equation is a matrix equation for the linear automorphisms of the tensor product of a vector space with itself. This equation plays a central role in quantum group theory and appears in many areas of mathematical physics. On the other hand, Hopf-Galois structures on Galois extensions of number fields are of great interest in Galois module theory as they reveal information about the rings of integers of these extensions. The classification of both the solutions Yang-Baxter equation and Hopf-Galois structures remain among important topics of research. In this talk we will explain how Hopf-Galois theory and the Yang-Baxter equation found to be related via algebraic objects known as skew braces. Then we will review our results on classification of skew braces of order p^3 , their automorphism groups, and their connection to Hopf-Galois structures.