SCHEDULE: HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 25–29, 2020

Half-baked ideas are welcome. This is a working conference. One of the features that everyone has enjoyed in previous conferences has been the inclusion of guiding principles, half-baked ideas, crazy conjectures. Please come with some to share.

Monday.

13:00GMT Caranti: *Hopf Galois structures, regular subgroups of the holomorph, and skew braces: two (brief) stories.* 50 min.

14:00GMT Nejabati Zenouz: *Skew Braces with Additive Group Isomorphic to a Semidirect Product of Cyclic Group of Order p with Cyclic Group of Order p^n and Corresponding Hopf-Galois Structures.* 50 min.

15:00GMT Stordy: *Fixed-Point-Free Abelian Endomorphisms, Braces, and the Yang-Baxter Equation.* 25 min.

15:40GMT Koch: *Abelian Maps, Braces, and Hopf-Galois Structures.* 50 min.

Tuesday.

13:00GMT Childs: *Existence theorems for Hopf Galois structures and skew braces: a survey.* 50 min.

14:00GMT Byott: *Counting Hopf-Galois Structures on Galois Extensions of Squarefree Degree and Skew Braces of Squarefree Order.* 50 min.

15:00GMT Kohl: *Mutually Normalizing Regular Permutation Groups and Zappa-Szep Extensions of the Holomorph.* 50 min.

Wednesday.

13:00GMT Underwood: *Hopf Forms and Hopf-Galois Theory.* 50 min.

14:00GMT Truman: *Conjugate Hopf-Galois structures.* 50 min.

15:00GMT Gil Muñoz: *A method to compute the associated order in extensions of p-adic fields.* 50 min.

Thursday.

13:00GMT Keating: *Generic p-extensions.* 50 min.

14:00GMT Moles: *Classifying totally ramified Galois extensions of prime power order over local fields.* 25 min.

14:40GMT Elder: *Beyond Heisenberg extensions: Ramification breaks in non-abelian extensions of degree p^n.* 50 min.

Abstracts

Nigel Byott, University of Exeter.

Counting Hopf-Galois Structures on Galois Extensions of Squarefree Degree and Skew Braces of Squarefree Order. 50 minutes

Abstract: Using the relationship between Hopf-Galois structures and skew braces, we will show, given two groups Γ, G of the same squarefree order n, how to determine

(a) the number $e(\Gamma, G)$ of Hopf-Galois structures of type G on a Galois extension of fields with Galois group isomorphic to Γ, and

(b) the number $b(\Gamma, G)$ of isomorphism classes of skew braces with multiplicative group isomorphic to Γ and additive group isomorphic to G.

The answer to (a) is rather involved, depending in an intricate way on the precise structure of Γ and G. The answer to (b) is much easier to state, but its proof depends upon the more complicated answer to (a).

(Joint work with Ali Alabdali, University of Mosul, Iraq.)

Andrea Caranti, Università degli Studi di Trento.

Hopf Galois structures, regular subgroups of the holomorph, and skew braces: two (brief) stories. 50 minutes

Abstract: Classifying skew braces with a given additive (not necessarily abelian) group $(G, \cdot)$ is the same as classifying the regular subgroups of the (permutational) holomorph of $(G, \cdot)$. This is in turn equivalent to classifying certain Hopf Galois structures.

In this talk we will discuss two topics.

- We will first consider a certain group that classifies the regular subgroups of the holomorph of a group G, which are isomorphic to G, and have the same holomorph as G.
  (This is joint work with Francesca Dalla Volta.)
- We will then describe a classification of the (conjugacy classes of) regular subgroups of the holomorph of a group G of order p^2q, where p and q are distinct odd primes, and the Sylow p-subgroups of G are cyclic.
  (This is joint work with Elena Campedel and Ilaria Del Corso.)

Lindsay Childs, University of Albany.

Existence theorems for Hopf Galois structures and skew braces: a survey. 50 minutes

Abstract: Call a pair of groups (Γ, G) realizable if there is a Γ-Galois extension of fields with a Hopf Galois structure of type G, or equivalently, there is a skew brace B with additive group isomorphic to G and adjoint

(circle) group isomorphic to Γ. There is an extensive literature in Hopf Galois theory (dating from 1996) and brace theory (dating from 1999) on the question, which pairs (Γ, G) are realizable? I will attempt to describe, or at least state, most of the known (to me, anyway) results on realizability. Included are results of Alebdali, Ault, Bachiller, Byott, Caranti, Childs, Crespo, Etinghof, Featherstonhaugh, Kohl, Kruse, Nasybullov, Rio, Rump, Salguero, Schedler, Smoktunowicz, Soloviev, Vela, Vendramin and Watters. Hopefully some will be new to much of the audience. And more hopefully, some of you can tell me about results I missed!

Griff Elder, University of Nebraska at Omaha.

Beyond Heisenberg extensions: Ramification breaks in non-abelian extensions of degree p^n. 50 minutes

Abstract: Let L/K be a totally ramified Galois extension of local number fields having residue characteristic p. When the Galois group $G = \mathrm{Gal}(L/K)$ is abelian, the ramification filtration of the Galois group is fairly well understood. Much remains unknown however, when the Galois group is nonabelian, even for small extensions. The Hasse-Arf Theorem no longer applies – upper ramification breaks need no longer be integers. And, as we saw last year in the case of characteristic p extensions with Galois group the Heisenberg group modulo p of order p^3, the ramification breaks can exhibit some interesting features. In this talk, we will explore a generalization of Heisenberg extensions – what I hope you will find to be an interesting family of characteristic p extensions of order p^n, $n \leq p$.

Daniel Gil Muñoz, Universitat Politècnica de Catalunya.

A method to compute the associated order in extensions of p-adic fields. 50 minutes

Abstract: Local Hopf Galois module theory seeks to describe, for an H-Galois extension L/K of local fields, both the structure of the associated order $\mathfrak{A}_H$ of the valuation ring $\mathcal{O}_L$ as $\mathcal{O}_K$-module and the freeness of $\mathcal{O}_L$ as $\mathfrak{A}_H$-module. In this talk we pick an extension of p-adic fields (i.e, local fields with characteristic 0) and we present a method to compute an $\mathcal{O}_K$-basis of $\mathfrak{A}_H$ which arises by regarding the action of H over L as a matrix, called the matrix of the action. We particularize to the case in which H is an induced Hopf Galois structure, that is, it may be described from Hopf Galois structures of more simple extensions. We present a sufficient condition under which $\mathfrak{A}_H$ splits in a similar way as H does.
(This is joint work with Anna Rio Doval.)

Kevin Keating, University of Florida.

Generic p-extensions. 50 minutes

Abstract: Let p be a prime and let G be a group of order p^n. In this talk I will discuss the existence of polynomials $D_i \in \mathbb{F}_p[X_1, \ldots, X_{i-1}]$ with the following property: For every field K_0 of characteristic p and every Galois extension K_n/K_0 with $\mathrm{Gal}(K_n/K_0) \cong G$ there are $\beta_i \in K_0$ such that K_n is constructed recursively by $K_i = K_{i-1}(\alpha_i)$, with $\alpha_i^p - \alpha_i = D_i(\alpha_1, .., \alpha_{i-1}) + \beta_i$. In addition, under mild assumptions on K_0 and β_i, every extension K_n/K_0 constructed this way is Galois, with $\mathrm{Gal}(K_n/K_0) \cong G$.
(This is joint work with Griff Elder.)

Alan Koch, Agnes Scott College.

Abelian Maps, Braces, and Hopf-Galois Structures. 50 minutes

Abstract: Let G be a finite group. In his 2013 paper *Fixed-point free endomorphisms and Hopf Galois structures*, Childs associates to any fixed-point free abelian endomorphism $\psi : G \to G$ an embedding $\alpha_\psi : G \hookrightarrow \mathrm{Perm}(G)$ whose image is regular and normalized by $\lambda(G)$ (which acts on $\mathrm{Perm}(G)$ via conjugation). This embedding gives (a) a Hopf-Galois structure on a Galois extension, group G; (b) a left skew brace; and (c) a set-theoretic solution to the Yang-Baxter Equation.

In this talk we will generalize Childs' result by eliminating the fixed-point free condition. We will show that an abelian endomorphism $\psi : G \to G$ gives a regular subgroup (of a potentially different permutation group), a Hopf-Galois structure, a left skew brace, and a solution to the YBE. We will show that Childs' construction coincides with ours when ψ is fixed-point free.

Tim Kohl, Boston University.

Mutually Normalizing Regular Permutation Groups and Zappa-Szep Extensions of the Holomorph. 50 minutes

Abstract: For a group G, embedded in its group of permutations $B = Perm(G)$ via the left regular representation $\lambda : G \to Perm(G)$, the normalizer of $\lambda(G)$ in B is $Hol(G)$, the holomorph of G. It is known that $Hol(G)$ is also the normalizer of $\rho(G)$ as well and both $\lambda(G)$ and $\rho(G)$ which are canonical examples of regular subgroups. The determination of those regular $N \leq Perm(G)$, where $N \cong G$ with the same normalizer is keyed to the structure of the so-called multiple holomorph of G, $NHol(G) = Norm_B(Norm_B(\lambda(G)))$. We wish to analyze the set of those subgroups of $Hol(G)$ which mutually normalize each other, but which don't necessarily have the same normalizer. This analysis gives rise to an object similarly containing $Hol(G)$, but

different than $NHol(G)$. We introduce what we call the quasi-holomorph of G which allows one to enumerate this set of mutually normalizing subgroups of $Hol(G)$. The quasi-holomorph will be group properly containing $Hol(G)$ and is frequently a Zappa-Szep product with the holomorph.

Grant Moles, University of Nebraska at Omaha (soon Clemson University).

Classifying totally ramified Galois extensions of prime power order over local fields. 25 minutes

Abstract: This presentation examines totally ramified Galois extensions of orderp^4, p an odd prime, over local fields. There are fifteen such Galois extensions, as presented by Burnside. Of the Galois groups of these extensions, five are abelian and ten are nonabelian. Only the nonabelian cases are handled here, since the abelian cases are trivial. These can be classified by determining the Artin-Schreier equations which determine the extensions. This is done by using the Weierstrass $\wp$-function, defined such that $\wp(x) = x^p - x$. Each nonabelian extension can be examined in this way, first by examining abelian and nonabelian groups of order p^3, then extending these to the desired extensions. Since the elements defining the extensions are non-unique, the definitions presented are non-unique; however, they are selected in such a way that the definitions of similar extensions are similar. Furthermore, the $\wp$-values of the defined extension elements are such that they each lie in a field extension over the base field of degree no more than p^2.

Kayvan Nejabati Zenouz, University of Greenwich.

Skew Braces with Additive Group Isomorphic to a Semidirect Product of Cyclic Group of Order p with Cyclic Group of Order p^n and Corresponding Hopf-Galois Structures. 50 minutes

Abstract: I will talk about the results of work in progress on classification of skew braces and Hopf-Galois structures whose type is isomorphic to $C_{p^n} \rtimes C_p$, for a prime number $p > 3$. I will describe the methods used in the classification of these skew braces and their automorphism groups. Consequently, I will use this classification to enumerate the Hopf-Galois structures parametrised by these skew braces, which correspond to all Hopf-Galois structures on a Galois extension of degree p^{n+1} whose type is a semidirect product of C_p by C_{p^n}.

Laura Stordy, Agnes Scott College (soon Georgia Tech).

Fixed-Point-Free Abelian Endomorphisms, Braces, and the Yang-Baxter Equation. 25 minutes

Abstract: Let ψ be a fixed-point-free abelian endomorphism on a finite group G. Using Childs' construction, we construct a regular, G-stable subgroup of $\text{Perm}(G)$ from ψ, which we denote N^ψ. In turn, we can find the brace corresponding to N^ψ and look at brace equivalences on these subgroups arising from fixed-point-free abelian endomorphisms. We construct all fixed-point-free abelian endomorphisms and corresponding braces on the symmetric group S_n, alternating group A_4, metacyclic group M_{pq}, and dihedral group D_n. Finally, we give the solutions to the set-theoretic Yang-Baxter equation that come from these braces.

Paul Truman, Keele University.

Conjugate Hopf-Galois structures. 50 minutes

Abstract: Let L/K be a Galois extension of fields with nonabelian Galois group G. If N is a regular subgroup of $\mathrm{Perm}(G)$ corresponding to a Hopf-Galois structure on L/K then N is normalized by $\lambda(G)$, but need not be normalized by $\rho(G)$. For each $\sigma \in G$, the group $N_\sigma = \rho(\sigma)N\rho(\sigma)^{-1}$ is regular and normalized by $\lambda(G)$, and so corresponds to a Hopf-Galois structure on L/K. We say that a Hopf-Galois structure arising in this way is *conjugate* to the Hopf-Galois structure corresponding to N.

We show that conjugate Hopf-Galois structures involve isomorphic Hopf algebras, correspond to the same skew left brace and, in the case that L/K is an extension of local or global fields, give "equally good" descriptions of the fractional ideals of L.

Rob Underwood, Auburn University at Montgomery.

Hopf Forms and Hopf-Galois Theory. 50 minutes

Abstract: Let K be a field containing $\mathbb{Q}$ and let C_n denote the cyclic group of order n. R. Haggenmüller and B. Pareigis have given a complete classification of all Hopf forms of KC_n for $n = 2, 3, 4, 6$. For all other n, the Hopf forms of KC_n seem difficult to compute. The linear dual $(KC_n)^*$ is a familiar Hopf form of KC_n, called the absolutely semisimple Hopf form. For $K = \mathbb{Q}$ we give an explicit description of $(\mathbb{Q}C_p)^*$ for p prime. We also discuss the connection between Hopf forms and Hopf-Galois theory.