

SCHEDULE: HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 24–28, 2021

Half-baked ideas are welcome! This has always been a working conference, and over the years, one of the most enjoyable features has been the inclusion of half-baked ideas, guiding principles, and crazy conjectures. Please come with some to share.

Zoom link: unomaha.zoom.us/j/99935837102?pwd=ZVRad1Qwb1JKR2ZkV2h5aU1jb1RxQT09

We intend to record the talks and post them on www.scm.keele.ac.uk/staff/p_truman/ConferenceArchive/.

If you would rather not have your talk recorded, please let me know.

Monday. Moderator: Paul Truman

- 13:00UTC** Byott: *Hopf-Galois structures on non-normal extensions of degree related to a Sophie Germain prime.* 50 min.
- 14:00UTC** Crespo: *Hopf Galois structures on separable field extensions of odd prime power degree.* 25 min.
- 14:30UTC** Colazzo: *Skew braces and solutions of the Yang-Baxter equation.* 50 min.
- 15:30UTC** Nejabati Zenouz: *Skew Braces with Additive Group Isomorphic to $C_{p^n} \times C_p$ and Corresponding Hopf-Galois Structures.* 50 min.

Tuesday. Moderator: Tim Kohl

- 13:00UTC** Koch: *Abelian maps and brace blocks.* 50 min.
- 14:00UTC** Caranti: *From endomorphisms to regular subgroups, (bi-)skew braces,* 25 min.
- 14:30UTC** Stefanello: *...the Yang-Baxter equation, and Hopf Galois structures.* 25 min.
- 15:00UTC** Underwood: *The Hagenmüller and Pareigis Bijection.* 50 min.

Wednesday. Moderator: Andrea Caranti

- 13:00UTC** Kohl: *Isomorphic Holomorphs and Bi-Skew Braces.* 50 min.
- 14:00UTC** Ezome: *Minimal Hopf-Galois structures on separable field extensions.* 25 min.
- 14:30UTC** Gil Muñoz: *Hopf-Galois module structure of quartic Galois extensions of \mathbb{Q} .* 50 min.
- 15:30UTC** Prestidge: *Hopf-Galois module structure of some non-normal extensions of number fields.* 50 min.

Thursday. Moderator: Nigel Byott

- 13:00UTC** Ferri: *Leopoldt-type theorems for non-abelian extensions of \mathbb{Q} .* 50 min.
- 14:00UTC** Del Corso: *How far is an extension of p -adic fields from having a normal integral basis?.* 25 min.
- 14:30UTC** Schwartz: *Galois scaffolds and Galois module structure for cyclic extensions of degree p^2 in characteristic 0.* 25 min.
- 15:00UTC** Keating: *The Hasse-Arf Theorem and Nonabelian Extensions.* 50 min.

ABSTRACTS

Nigel Byott, University of Exeter.

Hopf-Galois structures on non-normal extensions of degree related to a Sophie Germain prime. 50 minutes

Abstract: There has been much work on enumerating Hopf-Galois structures on Galois extensions, and this problem is well understood for extensions of squarefree degree. Less is known for extensions which are separable but not normal. I will report on joint work with Isabel Martin-Lyons and Andrew Darlington investigating Hopf-Galois structures on non-normal extensions L/K whose degree is a squarefree number of the very special form $n = pq$ for odd primes p, q with $p = 2q + 1$ (so q is a Sophie Germain prime and p is a safeprime). Working in terms of the permutation group given by G and G' , where $G = \text{Gal}(E/K)$, $G' = \text{Gal}(E/L)$ and E is the normal closure of L/K , we describe all extensions L/K of degree n which admit at least one Hopf-Galois structure, and find the number of Hopf-Galois structures in each case. In particular, we determine those cases where there are Hopf-Galois structures of both cyclic and non-abelian types. I will also present some preliminary results on the behaviour of other subfields of degree n inside E , and on possible generalisations.

Andrea Caranti, Università degli Studi di Trento.

From endomorphisms to regular subgroups, (bi-)skew braces, 25 minutes

Abstract: In his talk at Omaha 2020, Alan Koch has given a construction of bi-skew braces, starting with an endomorphism of a group whose image is abelian. Together with Lorenzo Stefanello, we have characterized the endomorphisms such that Koch's construction yields (bi-)skew braces.

In his talk at Omaha 2021, Koch shows how his approach can be used to construct, starting with an endomorphism of a group whose image is abelian, a brace block, that is, a possibly infinite set of group operations on the underlying set, so that the set becomes a skew brace with respect to any two such operations. We have extended Koch's construction by relaxing the assumption on the endomorphism, and allowing polynomials in the endomorphism.

Ilaria Colazzo, University of Exeter.

Skew braces and solutions of the Yang-Baxter equation. 50 minutes

Abstract: The Yang-Baxter equation, named after the authors of the seminal papers in which the equation arose, is a fundamental tool in several research fields such as statistical mechanics, quantum group theory, and low-dimensional topology. In 1992, Drinfel'd suggested focusing on set-theoretical solutions of the Yang-Baxter equation as a first step to studying solutions of the Yang-Baxter equation. These solutions were initially approached in a combinatorial fashion by Etingof, Soloviev and Schedler and Gateva-Ivanova and Van den Bergh. More recently, Rump introduced braces, an algebraic structure that generalises radical rings, to describe involutive non-degenerate solutions of the Yang-Baxter equation. To study non-involutive solutions, one needs skew braces, a non-commutative analogue of braces. In this talk, we briefly discuss

some basic properties of skew braces and how these structures are related to solutions of the Yang-Baxter equation. We also introduce a useful construction technique that allows one to characterise a subclass of skew braces.

(The talk will be mainly based on joint work with F. Catino and P. Stefanelli.)

Ilaria Del Corso, Università di Pisa.

How far is an extension of p -adic fields from having a normal integral basis? 25 minutes.

Abstract: Let L/K be a Galois extension with Galois group G . The Normal Basis Theorem shows that L is a free $K[G]$ -module of rank 1. When L/K is a number field or a local field extension, it is natural to consider the question of determining the structure of the ring of integers \mathfrak{O}_L as a $\mathfrak{O}_K[G]$ -module. It is well-known that \mathfrak{O}_L contains free $\mathfrak{O}_K[G]$ -submodules of finite index, but, in general, it is not free. In this talk, after a brief overview of the main classical results in this context, I will present some recent results on the minimal index of a free $\mathfrak{O}_K[G]$ -submodule into \mathfrak{O}_L , in the case when L/K is p -adic field extension.

(This is a joint work with F. Ferri and D. Lombardo.)

Teresa Crespo, Universitat de Barcelona.

Hopf Galois structures on separable field extensions of odd prime power degree. 25 minutes

Abstract: In this talk, we will present two results concerning Hopf Galois structures on a separable field extension L/K of degree p^n , for p an odd prime number. For $p > n$, we will see that L/K has at most one abelian type of Hopf Galois structures. This generalizes a result by Caranti, Childs and Featherstonhaugh for the Galois case. For our second result, given a nonabelian group N of order p^n , with commutator subgroup of order p , we may consider an abelian group A of order p^n and having the same number of elements of order p^m as N , for $1 \leq m \leq n$. We will show that if L/K has a Hopf Galois structure of type N , then it has a Hopf Galois structure of type A .

Tony Ezome, Université des Sciences et Techniques de Masuku (USTM).

Minimal Hopf-Galois structures on separable field extensions. 25 minutes

Abstract: Hopf-Galois theory generalizes the classical Galois theory. The concept of Hopf-Galois extension is due to Chase and Sweedler. They introduced it in 1969 to study purely inseparable extensions of fields and ramified extensions of rings. Then, Greither and Pareigis developed Hopf-Galois theory for separable field extensions in 1987. The first part of this talk is devoted to Hopf-Galois structures of separable field extensions. We will start from well known results within Galois theory and display remarkable differences between Galois and Hopf-Galois theories. Then, we will focus on Hopf-Galois structures which are minimal. All along the talk, we will give detailed examples.

Fabio Ferri, University of Exeter.

Leopoldt-type theorems for non-abelian extensions of \mathbb{Q} . 50 minutes

Abstract: We prove new results concerning the additive Galois module structure of certain wildly ramified finite non-abelian extensions of \mathbb{Q} . In particular, when K/\mathbb{Q} is a Galois extension with Galois group G isomorphic to A_4 , S_4 or A_5 , we give necessary and sufficient conditions for the ring of integers \mathfrak{O}_K to be free over its associated order in the rational group algebra $\mathbb{Q}[G]$.

Daniel Gil Muñoz, Universitat Politècnica de Catalunya.

Hopf-Galois module structure of quartic Galois extensions of \mathbb{Q} . 50 minutes

Abstract: A quartic Galois extension L/\mathbb{Q} of number fields can be either cyclic or biquadratic. By Leopoldt's theorem, the ring of integers \mathcal{O}_L is always a free $\mathfrak{A}_{L/\mathbb{Q}}$ -module, where $\mathfrak{A}_{L/\mathbb{Q}}$ is the associated order in the classical Galois structure of L/\mathbb{Q} . In this talk we explore whether \mathcal{O}_L is free over its associated order \mathfrak{A}_H in a non-classical Hopf Galois structure H of L/\mathbb{Q} , by examining directly the action of H on an integral basis of L . If L/\mathbb{Q} is cyclic, it has a single non-classical Hopf Galois structure and we obtain that \mathcal{O}_L is always \mathfrak{A}_H -free. If L/\mathbb{Q} is biquadratic, it has three non-classical Hopf Galois structures. In this case, we recover Truman's result on the conditions for the \mathfrak{A}_H -freeness of \mathcal{O}_L for tamely ramified biquadratic extensions and prove similar conditions for the wild case.

(This is joint work with Anna Rio)

Kevin Keating, University of Florida.

The Hasse-Arf Theorem and Nonabelian Extensions. 50 minutes

Abstract: Let K be a local field and let L/K be a finite Galois extension with Galois group $G = \text{Gal}(L/K)$. The Hasse-Arf theorem says that if G is abelian then every upper ramification break of L/K is an integer. This fact is crucial for explicating the relation between local class field theory and the ramification filtration of G . In this talk I will consider the problem of determining how often non-integral upper ramification breaks occur in nonabelian Galois extensions.

Alan Koch, Agnes Scott College.

Abelian maps and brace blocks. 50 minutes

Abstract: Let (G, \cdot) be a group, and let $\psi : G \rightarrow G$ be an endomorphism with abelian image. We show how ψ allows us to construct a sequence $\{\circ_m : m \in \mathbb{Z}^{\geq 0}\}$ of binary operations on G such that (G, \circ_m, \circ_n) is a (bi-skew) brace for all $m, n \geq 0$. This family of braces—which we call a *brace block*—gives a (potentially) large family of solutions to the Yang-Baxter equation which we can describe in terms of ψ . Additionally, each brace block allows us to construct a family of Hopf-Galois structures on Galois extensions with Galois group isomorphic to (G, \cdot) , or more generally for an extension with Galois group isomorphic to (G, \circ_n) for

any n . Examples will be given which will show that a brace block can include an arbitrarily large number of nonisomorphic braces.

Tim Kohl, Boston University.

Isomorphic Holomorphs and Bi-Skew Braces. 50 minutes

Abstract: For a given set X , which is a group with respect to two different operations (X, \star) and (X, \circ) , we consider the connection between (X, \star) and (X, \circ) having isomorphic holomorphs and the existence of bi-skew brace structures (X, \star, \circ) and (X, \circ, \star) .

George Prestidge, Keele University.

Hopf-Galois module structure of some non-normal extensions of number fields. 50 minutes

Abstract: Noether's theorem tells us that if L/K is a tame, Galois extension of number fields, with Galois group G , then \mathcal{O}_L is locally free over $\mathcal{O}_K G$. In general, criteria for global freeness are more delicate. Del Corso and Rossi gave such criteria for L/K a tame Kummer extension. We study a non-normal analogue of this situation using Hopf-Galois theory. Truman studied a family of non-normal, tame, radical extensions of prime degree. We generalise the work of Truman to certain families of tame extensions of prime-power degree which have a unique almost classical Hopf-Galois structure. We find that \mathcal{O}_L is locally free over its associated order in this Hopf-Galois structure and determine criteria for global freeness. These criteria are identical to those of Del Corso and Rossi in the Galois case.

Kayvan Nejabat Zenouz, University of Greenwich.

Skew Braces with Additive Group Isomorphic to $C_{p^n} \rtimes C_p$ and Corresponding Hopf-Galois Structures. 50 minutes

Abstract: I will talk about the results of work in progress on classification of skew braces and Hopf-Galois structures whose type is isomorphic to $C_{p^n} \rtimes C_p$, for a prime number $p > 3$. I will describe the methods used in the classification of these skew braces and their automorphism groups. Consequently, I will use this classification to enumerate the Hopf-Galois structures parametrised by these skew braces, which correspond to all Hopf-Galois structures on a Galois extension of degree p^{n+1} whose type is a semidirect product of C_p acting on C_{p^n} . In addition, I will briefly report on a recent new collaborative project in investigating the connection between braces and pre-Lie algebras.

Paul Schwartz, University of Florida.

Galois scaffolds and Galois module structure for cyclic extensions of degree p^2 in characteristic 0. 25 minutes

Abstract: Let K be a finite extension of \mathbb{Q}_p . We construct a family of cyclic extensions M/K of degree p^2 which possess a Galois scaffold. We then determine necessary and sufficient conditions for \mathfrak{D}_M to be free over its associated order. This parallels the work of Byott and Elder in 2013 who worked in the setting $\text{char}(K) = p$.

Lorenzo Stefanello, Università di Pisa.

... the Yang-Baxter equation, and Hopf Galois structures. 25 minutes

Abstract: In a joint work with Andrea Caranti, we have generalised some recent papers by Alan Koch concerning the construction of bi-skew braces, starting from an endomorphism of a group whose image is abelian. In his talk at Omaha 2021, Andrea Caranti presents some of these extended results, discussing the relation between endomorphisms, (bi)-skew braces, and regular subgroups. In this talk, we apply our findings to the study of the Yang-Baxter equation and Hopf Galois structures. In particular, we point out the role of gamma functions, which can be helpful in both settings.

Rob Underwood, Auburn University at Montgomery.

The Haggemüller and Pareigis Bijection. 50 minutes

Abstract: Let K be a finite field extension of \mathbb{Q} and let N be a finite group with automorphism group $F = \text{Aut}(N)$. R. Haggemüller and B. Pareigis have shown that there is a bijection

$$\Theta : \text{Gal}(K, F) \rightarrow \text{Hopf}(K[N])$$

from the collection of F -Galois extensions of K to the collection of Hopf forms of the group ring $K[N]$. For $N = C_n$, $n \geq 1$, $N = C_p^m$, p prime, $m \geq 1$, and $N = D_3, D_4, Q_8$, we show that $\mathbb{Q}[N]$ admits an absolutely semisimple Hopf form H and find L for which $\Theta(L) = H$. Moreover, if H is the Hopf algebra given by a Hopf-Galois structure on a Galois extension E/K , we show how to construct the preimage of H under Θ assuming certain conditions.

(This is joint work with Tim Kohl.)