HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 28 – 31, 2024

Half-baked ideas are welcome! This has always been a working conference, and over the years, one of the most enjoyable features has been the inclusion of half-baked ideas, guiding principles, and crazy conjectures. Please come with some to share.

Break-out rooms provided. Another valuable feature of this conference has always been the ample time provided for discussion. Zoom break-out rooms will be available after every two talks. One room named for the first talk, one for the second, and one for general conversation.

Zoom link: `https://unomaha.zoom.us/j/95655932060?pwd=SGhjTTczb29NVTF2QXowclFaYnVBQT09`
We intend to record the talks and post them at `www.hopf-galois.org/`. If you would rather not have your talk recorded, please let me know.

Time Zone Conversion. You might find this helpful. I did.

CDT	EDT	**UTC**	BST	CEST	JST
8:00	9:00	**13:00**	14:00	15:00	22:00
12:00	13:00	**17:00**	18:00	19:00	2:00

Tuesday. Moderator: Tim Kohl

13:00UTC Cindy (Sin Yi) Tsang, *Maschke's theorem and irreducible representations for skew braces.* 25 minutes
13:30UTC Paul Truman, *Skew bracoids and the Yang-Baxter equation.* 50 minutes
BREAK 14:30UTC – Break-out rooms available.
15:00UTC Alan Koch, *Skew left bracoids via abelian maps.* 25 minutes
15:30UTC Isabel Martin-Lyons, *Almost Classical Skew Bracoids.* 25 minutes
BREAK 16:00UTC – Break-out rooms available.

Wednesday. Moderator: Kevin Keating

13:00UTC Nigel Byott, *Counting Quaternion and Dihedral Braces and the Associated Hopf-Galois Structures.* 25 minutes
13:30UTC Tim Kohl, *Isomorphic Holomorphs.* 50 minutes.
BREAK 14:30UTC – Break-out rooms available.
15:00UTC Teresa Crespo, *Left braces of size $p^2(2p+1)^2$, for p an odd Germain prime.* 25 minutes.
15:30UTC Robert Underwood, *The Baer Product and Extensions of Hopf Orders.* 50 minutes.
BREAK 16:30UTC – Break-out rooms available.

Thursday. Moderator: Robert Underwood

13:00UTC Kevin Keating, *Abrashkin's work on the higher ramification filtration I.* 50 minutes
14:00UTC Paul Schwartz, *A Heisenberg description of nilpotent groups of class* 2 *and progress toward Galois scaffolds.* 25 minutes
BREAK 14:30UTC – Break-out rooms available.
15:00UTC Ilaria Del Corso, *The (Hopf-)Galois module structure of integers in radical extensions I.* 25 minutes
15:30UTC Paul Truman, *The (Hopf-)Galois module structure of integers in radical extensions II.* 25 minutes
BREAK 16:00UTC – Break-out rooms available.

6:00PM CDT Dinner at *the one and only* Johnny's Cafe, 4702 S 27th St. 1922 chophouse with a throwback feel highlighting Midwest beef, including steaks aged on-site.

Friday. Moderator: Paul Truman

13:00UTC Kevin Keating, *Abrashkin's work on the higher ramification filtration II.* 50 minutes
BREAK 14:00UTC – Break-out rooms available.
14:30UTC Lorenzo Steffanello, *Classifying Galois extensions with Childs's property.* 25 minutes
15:00UTC Calum Heldt, *Artin-Schreier presentations for nonabelian extensions of degree p^5.* 25 minutes
BREAK 15:30UTC – Break-out rooms available.

Abstracts

Nigel Byott, University of Exeter
Counting Quaternion and Dihedral Braces and the Associated Hopf-Galois Structures.
25 minutes
N.P.Byott@exeter.ac.uk
Coauthors: Fabio Ferri

Abstract: In the 2017 paper where they introduced skew braces, Guarnieri and Vendramin presented the results of computer calculations enumerating braces, and on the basis of these made several conjectures. One of these predicts for all $m > 2$ the number $q(4m)$ of isomorphism classes of braces of size $4m$ whose multiplicative group is a generalised quaternion group. Rump (2020) gave a proof of this in the case $m = 2^n$, $n \geq 3$. I will outline a proof of this conjecture for all m, together with the analogous result for dihedral braces. The methods used have their origin in Hopf-Galois theory, and we also obtain results on the number of Hopf-Galois structures on a Galois extension of fields with quaternion or dihedral Galois group.

Teresa Crespo, Universitat de Barcelona
Left braces of size $p^2(2p+1)^2$, for p an odd Germain prime.
25 minutes.
teresa.crespo@ub.edu

Abstract: In my talk I shall consider braces of size mn, where m and n are relatively prime integer numbers such that each group of order mn has a normal subgroup of order m. I shall prove that any such brace is a semidirect product of a brace of size m and a brace of size n and describe precisely how the classification of braces of size mn follows from the classification of braces of sizes m and n. I shall present the application of the previous result to determine all braces of size p^2q^2, for p an odd Germain prime and $q = 2p + 1$.

Ilaria Del Corso, Università di Pisa
Paul Truman, University of Keele
The (Hopf–)Galois module structure of integers in radical extensions.
25 + 25 minutes
ilaria.delcorso@unipi.it
p.j.truman@keele.ac.uk
Coauthors: Lorenzo Rossi

Abstract: The question of the existence of a normal integral basis in the case of Galois extensions of number fields, or its analogue for general extensions in the Hopf–Galois context, is old but still largely open. In this talk, we survey the state of the art for Kummer, and more generally, radical extensions.

In the first half of this talk, we consider the general case of tamely ramified Kummer extensions and give a criterion for the existence of an NIB. We also explicitly describe the Steinitz class of a tame Kummer extension, providing an easy criterion for this class to be trivial.

In the second half, we consider generalisations of these ideas to non-normal tamely ramified radical extensions of number fields, using almost classical Hopf-Galois structures. We exhibit families of examples in which the ring of integers is locally free over its associated order and the criteria for global freeness are identical to those in the Galois case.

Calum Heldt, University of Nebraska at Omaha
Artin-Schreier presentations for nonabelian extensions of degree p^5.
25 minutes
cheldt@unomaha.edu

Abstract:

Kevin Keating, University of Florida
Abrashkin's work on the higher ramification filtration
50 minutes
keating@ufl.edu

Abstract: In these two talks I will describe Abrashkin's characterization of the higher ramification subgroups of certain Galois extensions of local fields of characteristic p. In the first talk I will provide some background and describe the tools that are needed to understand Abrashkin's work in this area. In the second talk I will give an overview of Abrashkin's characterization and work out some examples.

Alan Koch, Agnes Scott College
A construction of skew bracoids with a single group
25 minutes
akoch@agnesscott.edu

Abstract: Let G be a group. We give a construction of bracoids $(G, \cdot, H, \cdot, \odot)$ with $H \leq G$. This construction is a generalization of a special case of bracoids developed by the author and Paul Truman, presented at Omaha in 2023, which depend on a certain endomorphism $\psi : G \to G$. Under certain circumstances, these bracoids give left degenerate, right non-degenerate solutions to the Yang-Baxter equation.

Timothy Kohl, Boston University
Isomorphic Holomorphs
50 minutes
tkohl@bu.edu

Abstract: For a finite group G, one defines the holomorph to be the normalizer $Norm_B(\lambda(G))$ in $B = Perm(G)$ of the left regular representation, $\lambda(G) \leq B$ which, as an abstract group, is isomorphic to $G \rtimes Aut(G)$. More broadly, any group can be regularly embedded as a subgroup of some symmetric group $B = Perm(X)$ and the corresponding normalizer can be computed. For two groups of the same order, we consider when they have isomorphic holomorphs, and more broadly when two groups of the same order have regular representations in some common ambient symmetric group, whose normalizers are equal. The prototype examples of this are D_{2n} and Q_n, the dihedral and quaternion groups of order $4n$. We consider the subtle distinctions between the isomorphism of their holomorphs as abstract groups, versus the equality of the normalizers of these regular representations. In particular we consider the possibility that two groups of the same order can have isomorphic abstract holomorphs but where there may not exist regular representations with common normalizer. We also consider connections with group presentations, Hopf-Galois structures, and skew braces.

Isabel Martin-Lyons, Keele University
Almost Classical Solutions to the Yang-Baxter Equation
25 minutes
i.d.martin-lyons@keele.ac.uk

Abstract: The skew bracoid is an algebraic structure which corresponds with Hopf-Galois structures on separable, but not necessarily normal, extensions of fields. Recently, we have seen a connection between skew bracoids and solutions to the set-theoretic Yang-Baxter equation, requiring some technical property of the skew bracoid. A natural case to investigate from a Hopf-Galois theory perspective is that of almost classical extensions and further almost classical structures. We specify to this almost classical case, recalling what this means for a skew bracoid, showing that all almost classical skew bracoids satisfy the condition necessary to produce a solution and finally investigating the solutions that arise.

Paul Schwartz, Stevens Institute of Technology
A Heisenberg description of nilpotent groups of class 2 *and progress toward Galois scaffolds.*
25 minutes
pschwart@stevens.edu

Abstract: We look at a relatively new result by Szabó which describes finite nilpotent groups of class at most two as subgroups of (poloarized) Heisenberg groups. In 2022, Keating and S constructed generalized Heisenberg extensions of local fields which posses a Galois scaffold. Using these results we construct class 2 nilpotent extensions of exponent p of local fields in characteristic 0. Given such an extension L/K, we work toward finding sufficient conditions for L/K to posses a Galois scaffold. Ultimately, when L/K is such an extension that posses a Galois scaffold, we use the theory of Byott, Childs, and Elder to give sufficient conditions for $\mathfrak{O}_L$ to be free over its associated order $\mathfrak{A}_{L/K} = \{\alpha \in K[\mathrm{Gal}(L/K)] : \alpha\mathfrak{O}_L \subseteq \mathfrak{O}_L\}$.

Lorenzo Stefanello, Università di Pisa
Classifying Galois extensions with Childs's property
25 minutes
lorenzo.stefanello@phd.unipi.it
Coauthors: Senne Trappeniers

Abstract: The transition from Galois theory to Hopf-Galois theory introduces complexities. One example is the potential lack of bijectivity of the Hopf-Galois correspondence, which implies that in principle the fundamental theorem of Galois theory can only be recast in a weaker version for Hopf-Galois structures.

In 2017, Childs proved that every Hopf-Galois structure on a cyclic Galois extension of odd prime power degree has a bijective Hopf-Galois correspondence. The aim of this talk is to provide a complete classification of Galois extensions that exhibit this property. We achieve this by employing a description of Hopf–Galois structures via skew braces, as outlined in a recent joint work with Trappeniers.

Paul Truman, Keele University
Skew bracoids and the Yang-Baxter equation
50 minutes
P.J.Truman@Keele.ac.uk
Coauthors: Ilaria Colazzo, Alan Koch, Isabel Martin-Lyons

Abstract: Braces and skew braces were introduced to provide an algebraic framework for studying bijective nondegenerate solutions of the set theoretic Yang-Baxter equation, and have subsequently been found to correspond with Hopf-Galois structures on finite Galois extensions. This correspondence can be extended to the case of finite separable, but potentially non-normal, extensions via *skew bracoids*. In this talk we investigate connections between skew bracoids and the set theoretic Yang-Baxter equation. We show that left skew bracoids satisfying a mild technical hypothesis can be used to obtain right nondegenerate solutions, study properties of these solutions, and give examples arising from various methods of constructing left skew bracoids.

Cindy (Sin Yi) Tsang, Ochanomizu University, Tokyo
Maschke's theorem and irreducible representations for skew braces
25 minutes
tsang.sin.yi@ocha.ac.jp
Coauthors: Yuta Kozakai

Abstract: A representation of a skew brace $A = (A, \cdot, \circ)$ is a triplet (β, ρ, V), where

$$\beta : (A, \cdot) \to \mathrm{GL}(V), \quad \rho : (A, \circ) \to \mathrm{GL}(V)$$

are representations of the additive and multiplicative groups of A, respectively, on the same vector space V such that

$$\beta((a \circ b) \cdot a^{-1}) = \rho(a)\beta(b)\rho(a)^{-1}$$

holds for all $a, b \in A$. The meaning of this compatibility condition will be explained in the talk. We prove the analog of Maschke's theorem for skew brace representations. We also give some explicit examples to illustrate that irreducible representations of skew braces seem to be much more difficult to understand that those of groups. This is joint work with Y. Kozakai.

Robert Underwood, Auburn University at Montgomery
The Baer Product and Extensions of Hopf Orders
50 minutes

runderwo@aum.edu

Abstract: Let p be prime and let K be a field of characteristic p that is complete with respect to a discrete valuation. Let R denote the valuation ring. Let C_p^n denote the elementary abelian group of order p^n and let C_{p^n} be the cyclic group of order p^n for $n = 1, 2, 3$. For $i_1, i_2 \geq 0$ integers, let $E(i_1), E(i_2)$ be Hopf orders in $K[C_p]$, and let $\mathcal{E}(E(i_2), E(i_1))$ denote the group of equivalence classes of short exact sequences of Hopf orders

$$R \longrightarrow E(i_1) \longrightarrow H \longrightarrow E(i_2) \longrightarrow R.$$

We may endow $\mathcal{E}(E(i_2), E(i_1))$ with the Baer product. G. G. Elder and U (2017) have classified the subgroup $\mathcal{E}_{gt}(E(i_2), E(i_1))$ of $\mathcal{E}(E(i_2), E(i_1))$, consisting of the generically trivial extensions.

Let A be an R-Hopf order in $K[C_{p^2}]$, inducing the short exact sequence of R-Hopf orders

$$E: \ R \longrightarrow E(i_1) \longrightarrow A \longrightarrow E(i_2) \longrightarrow R.$$

Necessarily, $pi_2 \leq i_1$, and so, there is a distinguished extension E_0 for which

$$[E] = [E_0] * [E_\mu],$$

where $*$ denotes the Baer product. In this way we compute A, and thus recover a result of D. Tossici (2010).

Extending to the $n = 3$ case, let $E(i_1, i_2, \mu)$ be an R-Hopf order in $K[C_p \times C_p]$ and let $E(i_3)$ be an R-Hopf order in $K[C_p]$. U (2022) has classified the generically trivial extensions $\mathcal{E}_{gt}(E(i_3), E(i_1, i_2, \mu))$. The middle terms of these extensions are Hopf orders in $K[C_p^3]$. We use the Baer product to compute extensions whose middle terms are Hopf orders in $K[C_{p^2} \times C_p]$ or $K[C_{p^3}]$.