Hopf Algebras and Galois Module Theory Conference: Schedule and Abstracts

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There is no waiting room. The passcode is hopf26.

EDT	UTC	
EDT	**UTC**	**Monday, 25 May.** Moderator: Alan Koch
9:00	**13:00**	Tim Kohl, Boston University. *Regular quasigroup actions and Zappa-Szep products* (50 minutes)
10:00	**14:00**	Cornelius Greither, Universität der Bundeswehr München. *Infinite-dimensional Hopf-Galois extensions, and a new(?) look at the HG correspondence* (25 minutes)
10:30	**14:30**	Break
11:00	**15:00**	Andrew Darlington, Vrije Universiteit Brussel. *Hopf-Galois structures of cyclic type on parallel extensions of prime power degree* (50 minutes)
EDT	**UTC**	**Tuesday, 26 May.** Moderator: Paul Truman
9:00	**13:00**	Daniel Gil-Muñoz, Universitat de Barcelona. *Hopf-Galois module structure of monogenic cubic number fields* (50 minutes)
10:00	**13:30**	Hal Simpson, University of Leeds. *Bicyclic biskew braces* (25 minutes)
10:30	**14:30**	Break
11:00	**15:00**	Kevin Keating, University of Florida. *Computing local Galois module structure* (50 minutes)
EDT	**UTC**	**Wednesday, 27 May.** Moderator: Tim Kohl
9:00	**13:00**	Paul Truman, Keele University. *Classifying bidihedral skew braces part 1: constraints and necessary conditions* (50 minutes)
10:00	**14:00**	Ansley Rickson, Agnes Scott College. *Dynamic irreducibility over finite fields* (25 minutes)
10:30	**14:30**	Break
11:00	**15:00**	Alan Koch, Agnes Scott College. *Classifying bidihedral skew braces part 2: the classification* (50 minutes)
EDT	**UTC**	**Thursday, 28 May.** Moderator: Kevin Keating
9:00	**13:00**	Maria Ferrara, Università degli Studi della Campania. *A Sylow theorem for some classes of finite skew braces* (50 minutes)
10:00	**14:00**	Massimiliano Di Matteo, Università degli Studi della Campania. *Chain conditions on skew braces and solutions of the Yang-Baxter equation* (25 minutes)
10:30	**14:30**	Break
11:00	**15:00**	Ilaria Colazzo, University of Leeds. *Let's weld: skew braces and welded Yang-Baxter solutions* (25 minutes)
11:30	**15:30**	Robert Underwood, Auburn University at Montgomery. *Isomorphism classes of Galois extensions* (50 minutes)
EDT	**UTC**	**Friday, 29 May.** Moderator: Robert Underwood
9:00	**13:00**	Cindy Tsang, Ochanomizu University. *Minimal Hopf-Galois structures on Galois extensions* (25 minutes)
9:30	**13:30**	Marco Trombetti, Università degli Studi Napoli Federico II. *On Dedekind skew braces* (50 minutes)
10:30	**14:30**	Break
11:00	**15:00**	Leandro Vendramin, Vrije Universiteit Brussel. *Rubik's as Galois'* (50 minutes)

Abstracts

I. Colazzo. *Let's weld: skew braces and welded Yang–Baxter solutions* (25 minutes)

Skew braces are a powerful algebraic source of set-theoretic solutions to the Yang–Baxter equation. On the other hand, welded braid groups arise naturally in low-dimensional topology, for instance as motion groups of unlinked circles in three-space, and welded solutions give rise to colouring invariants of welded knots and links.

In this talk, I will discuss how these two worlds meet. Starting from the solution associated to a skew brace, I will ask when it is compatible with the twist solution in the welded sense. This leads to a notion of welded skew brace. I will present a characterisation: a skew brace is welded precisely when it is bi-skew, together with an additional commutativity condition on the right action. I will illustrate the result through examples, including braces arising from nilpotent rings and from group actions, and explain how these recover and generalise known welded biquandle constructions.

This is joint work with João Faria Martins, Victoria Lebed, and Senne Trappeniers.

A. Darlington. *Hopf–Galois structures of cyclic type on parallel extensions of prime power degree* (50 minutes)

Given a separable extension L/K of degree n with Galois closure E, a *parallel extension* to L/K is any degree n subextension of E/K. We can then ask the following question of any such parallel extension L'/K to L/K:

If L/K admits a Hopf–Galois structure of type N, when does L'/K also admit a Hopf–Galois structure of type N?

In this talk, we will answer this question fully in the case that N is a cyclic group of prime power order (for both odd and even primes). This talk is based on joint work with Cindy Tsang.

Massimiliano Di Matteo. *Chain Conditions on Skew Braces and Solutions of the Yang-Baxter Equation* (25 minutes)

The Yang–Baxter Equation (YBE) is a fundamental equation in quantum and statistical mechanics, and the study of its solutions has become a point of convergence across different disciplines. In particular, the study of its set-theoretical solutions has led to skew braces, a new algebraic structure with a dual nature oscillating between groups and rings.

Set-theoretical solutions are associated with infinite skew braces, and therefore, it is natural to analyse chain conditions in skew braces: maximal and minimal conditions on subbraces and ideals. We will present their interrelations within the framework of the structure theory of skew braces. We relate our results to classical results in the theory of chain conditions in groups. Additionally, chain conditions are also defined to deal with infinite set-theoretical solutions. These could play a decisive role in the analysis of solutions to the equation in infinite-dimensional vector spaces.

This is a joint work with Ramon Esteban-Romero, Maria Ferrara and Vicent Pérez-Calabuig.

Maria Ferrara. *A Sylow Theorem for Some Classes of Finite Skew Braces* (50 minutes)

We study a skew brace analogue of the First Sylow Theorem for finite groups. Although a general version of this result is not yet available in the context of skew braces, we show that it holds for several notable classes of finite skew braces.

This work is carried out in collaboration with Andrea Caranti, Ilaria Del Corso, Massimiliano Di Matteo, and Marco Trombetti.

D. Gil-Muñoz. *Hopf-Galois module structure of monogenic cubic number fields* (50 minutes)

Let L be a cubic number field. It is known that the extension $L/\mathbb{Q}$ possesses a primitive element α which is a root of a polynomial of the form $x^3 - ax + b$, where $a, b \in \mathbb{Z}$ are such that $v_p(a) < 2$ or $v_p(b) < 3$ for each prime number p. In this talk, we shall discuss the problem of the freeness of the ring of integers $\mathcal{O}_L$ as a module over its associated order $\mathfrak{A}_H$ in the unique Hopf-Galois structure H on $L/\mathbb{Q}$. This problem is approached by means of the method introduced by Rio in the author to produce a $\mathbb{Z}$-basis of $\mathfrak{A}_H$. In order to apply this method, the knowledge of an integral basis for L is needed, and this is done through the work by Alaca on p-integral bases of cubic number fields. A complete solution for the $\mathfrak{A}_H$-freeness of $\mathcal{O}_L$ is presented under the hypothesis that $\mathcal{O}_L = \mathbb{Z}[\alpha]$.

C. Greither. *Infinite-dimensional Hopf-Galois extensions, and a new(?) look at the HG correspondence* (25 minutes)

If a Hopf algebra H acts or coacts on a commutative K-algebra A, then A is Hopf Galois for H iff a certain map constructed from the structural map defining the (co-)action is bijective. This definition comes in two versions, for actions (H-extensions, A is an H-module) and for co-actions (H-objects, A is an H-comodule), and when the algebras are finite-dimensional, the two versions are equivalent via a dualizing argument, where H becomes H^* on the comodule side. In a recent paper, Bui, Vercruysse and Wiese exhibit a similar description (a certain derived map has to be bijective) also for subalgebras B that arise in the Hopf Galois correspondence; note that this is now a result, not a definition. They only do it on the module side, but they include an infinite-dimensional version, which is well adapted to number theory; there H needs to be given a certain topology, called pro-artinian. In the present talk we intend to show that this bijectivity criterion has a parallel on the comodule side, and this is useful for the simple reason that on that side no topology is required on any object. We moreover explain the translation into the language of Γ-sets and Γ-groups, where Γ is the absolute Galois group of the base field K. This is not new, but we think that the resulting description of the Hopf Galois correspondence is simple and nice. Concerning all aspects of the talk, the speaker wonders how much of this is perhaps already known, and hopes for enlightening comments from the audience.

K. Keating. *Computing local Galois module structure* (50 minutes)

In this talk I will consider the problem of computing the local Galois module structure of ideals in extensions of local fields. I will focus on Leopoldt's question, which asks whether an ideal is free over its associated order.

A. Koch. *Classifying bidihedral skew braces part II: the classification* (50 minutes)

In part I we presented necessary conditions for a skew left brace $(G, \cdot, \circ)$ to be bidihedral, that is, for $(G, \cdot)$ and $(G, \circ)$ to be dihedral groups. In this part we explicitly construct all bidiheral braces.

Given the strong connections between skew left braces and Hopf-Galois theory on separable extensions, we relate our classification to T. Kohl's classification of Hopf-Galois structures on a dihedral Galois extension.

(Joint work with P. Truman)

T. Kohl. *Regular Quasigroup Actions and Zappa-Szep Products* (50 minutes) By work of Neumann and Miller, it is a known, but somewhat unexplored, fact that Zappa-Szep products of groups correspond to transitive group actions where the two components correspond to a point stabilizer, and a subgroup acting regularly. From the other direction, one may consider an arbitrary transitive group G, and seek to find a decomposition of it into a product RF, where F is any point stabilizer and R is a putative regular subgroup. It turns out however, that there exist transitive permutation groups, which are termed *elusive*, which have no regular subgroups. In this situation however, we consider a generalization by instead looking at subsets of G which are quasigroups that act in a way analogous to regularity. We consider the distinction between this construction, and Miller/Neumann's results, and what it implies about transitive groups which one might expect to be Zappa-Szep products of this type, but are actually elusive.

A. Rickson. *Dynamical Irreducibility Over Finite Fields* (25 minutes)

Arithmetic dynamics seeks to understand and classify dynamical behavior of rational maps over fields of number theoretic interest like finite fields. Over finite fields, one may think the dynamical behavior of a polynomial would be simple, but that is not always the case. If all iterates of a polynomial are irreducible, we say the polynomial is *dynamically irreducible*. This work looks closely at the dynamical irreducibility of polynomials of the form $ax^d + c$ where a and c are elements of a finite field and d is a power of 3, revealing patterns among dynamically irreducible pairs (a, c) and further increasing knowledge about dynamical irreducibility.

H. Simpson. *Bicyclic Biskew Braces* (25 minutes)

Skew braces are an algebraic structure with links to Hopf-Galois theory. Bicyclic skew braces were first classified by Rump via cocyclic residue classes. We instead use results from Hopf-Galois theory to classify all bicyclic skew braces of finite order, and among them which are biskew.

M. Trombetti. *On Dedekind skew braces* (50 minutes)

In general, (finite) skew braces may have very few proper non-zero sub-skew braces, and this poses a major difficulty in understanding their global structure. Indeed, one must often rely on the definition of the operations themselves, which can be difficult to handle. The aim of this talk is to present a study of the class of skew braces in which, although proper non-zero sub-skew braces may (in principle) be scarce, they are always ideals. As we will see, this additional requirement either yields a rich supply of sub-structures or enables a satisfactory description even when only a few exist.

P. Truman. *Classifying bidihedral skew braces part 1: constraints and necessary conditions* (50 minutes)

We study finite skew braces $(G, \cdot, \circ)$ in which $(G, \cdot)$ and $(G, \circ)$ are both dihedral groups. We find that the cyclic subgroup $(H, \cdot)$ of $(G, \cdot)$ consisting of all rotations is an ideal; hence $(H, \circ)$ is also a subgroup of $(G, \circ)$, and so $(H, \circ)$ is either cyclic or dihedral.

Skew braces $(H, \cdot, \circ)$ in which $(H, \cdot)$ and $(H, \circ)$ are both cyclic are classified by Rump; those in which $(H, \cdot)$ is cyclic and $(H, \circ)$ is dihedral are classified by Byott and Ferri. Building on these works, we obtain constraints on the structure of the skew brace $(G, \cdot, \circ)$.

(Joint work with A. Koch.)

C. Tsang. *Minimal Hopf–Galois structures on Galois extensions* (25 minutes)

Let L/K be a finite extension of fields. For any Hopf–Galois structure H on L/K, the Hopf–Galois correspondence Φ_H, from the K-Hopf subalgebras of H to the intermediate fields of L/K, is always injective but need not be bijective in general. One can ask when Φ_H is bijective, but in this talk, we consider the other extreme and ask when Φ_H is as far from being bijective as possible. More specifically, when $[L : K] \geq 2$, we say that H is a *minimal* Hopf–Galois structure if the image of Φ_H contains only K and L. We give a partial classification, in terms of skew braces, of the minimal Hopf–Galois structures on Galois extensions.

R. Underwood. *Isomorphism classes of Galois extensions* (50 minutes)

Let F be a finite group, let K be a field and let A be an F-Galois extension of K in the sense of Chase, Harrison, Rosenberg. Then A determines a subgroup $U \leq F$ and a Galois field extension L/K with group U. Moreover, given a subgroup $U \leq F$ and a Galois field extension L/K with group U, we construct an F-Galois extension $A(U, L/K)$ using the induction map of Pareigis. In fact, every F-Galois extension A can be written as $A = A(U, L/K)$ for some $U, L/K$.

Suppose $U, V \leq F$, with U, V distinct and isomorphic. Suppose that L/K is Galois with group U and with group V. We prove the following: U is conjugate to V if and only if $A(U, L/K) \cong A(V, L/K)$ as F-Galois extensions of K. We show how this result relates to the Haggenmüller-Pareigis bijection and give some examples.

L. Vendramin. *Rubik's as Galois'* (50 minutes)

The Inverse Galois Problem asks whether every finite group occurs as a Galois group over the rationals. In this talk, I will first introduce the Inverse Galois Problem, discussing its history, main challenges, and some classic developments. I will then turn to the group theory of the Rubik's Cube and prove that the Rubik's Cube group is a Galois group. Finally, I will discuss how other puzzle groups can be realized as Galois groups. The talk is based on joint works with Martín Mereb.